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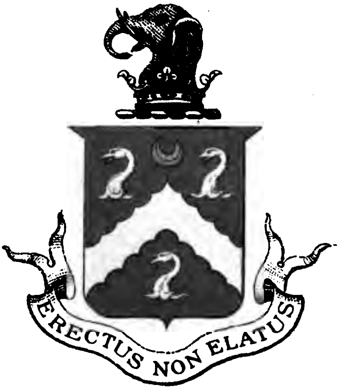

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NOTES AND PROBLEMS

ON THE

ELEMENTS OF MECHANISM

AND THE

TRANSMISSION OF POWER

COMPILED AND PREPARED

FOR THE USE OF

STUDENTS IN STEAM AND MECHANISM.

UNITED STATES ARTILLERY SCHOOL,

FORT MONROE, VA.

BY

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PREFACE.

These notes have been compiled from the works of Rankine, Schwamb, and Goodeve, with additions from various sources, for the convenience of students at the Artillery School in order to avoid a multiplicity of text books, and to present in a concise form, principles and facts concerning the transmission of power, that may be of interest and benefit tothe artillerist. The works quoted are

Rankine's "The Steam Engine."
Rankine's "Machinery and Mill work."
Schwamb's notes on the Elements of Mechanism,
    and notes on Toothed Wheels and Gearing.
Goodeve's "Principles of Mechanics."
Goodeve's "Elements of Mechanism."
The catalogues of the Page Link Belting Co., and
    of Brown and Sharp.

W. B. H.

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NOTES ON MECHANISM.

INTRODUCTORY.

The science of Mechanism treats of the designing and construction of machinery.

1. A MACHINE is a combination of resistant bodies so arranged that by their means the mechanical forces of nature can be compelled to produce some effect or work accompanied with certain determinate motions. In general, it may be properly said that a machine is an assemblage of moving parts interposed between the source of power and the work, for the purpose of adapting the one to the other.

No machine can move itself, nor can it create any motive power, this must be derived from external sources, such as the force of gravitation, the uncoiling of a spring, or the expansion of steam. As an example of a machine commonly met with, a sewing machine might be cited. It is capable of doing work of a definite kind, provided some external source of energy shall act upon it and set the working parts in motion. On examining its construction, we should find a fixed framework supporting combinations of movable parts; some of which are employed in actuating a needle and shuttle, while others carry forward the material to be stitched. The moving parts are so arranged that they must make certain definite motions relatively to each other, when some natural force, as the power of the hand or foot or of a rotating shaft, is applied to the proper recipient; then the work is done as a necessary consequence of the action of the motive power.

The operation of any machine depends upon two things: first the transmission of certain forces, and second, the production of determinate motions. In designing, due consideration must be given both of these, so that each part may be adapted to bear the strains imposed on it, as well as have the proper relative motion in regard to the other parts of the machine.

But the nature of the movements does not depend upon the strength or absolute dimensions of the moving parts, as can be shown by models whose dimensions may vary from those requisite for strength, and yet the motions of the parts will be the same as those of the machine. Therefore, the *force* and the *motion* may be considered separately, thus dividing the science of Mechanism into two parts, viz.:—

1. PURE MECHANISM, which treats of the motions and forms of the parts of a machine, and the manner of supporting and guiding them, independent of their strength.

2. CONSTRUCTIVE MECHANISM, which involves the calculation of the forces acting on the different parts of the machine; the selection of materials as to strength and durability in order to withstand the forces, taking into account the convenience for repairs, and facilities for manufacture.

In what follows, we will, in general, confine ourselves to the first part, *pure mechanism*, or what is sometimes called "the geometry of machinery", but will in some cases consider the forces in action.

Then our definition of a machine might be modified to accord with the above, as follows:—

2. A MACHINE is an assemblage of moving parts so connected that when the first, or recipient, has a certain motion, the parts where the work is done, or effect produced, will have certain other definite motions. *A Mechanism* is a term applied to a portion of a machine where two or more pieces are combined, so that the motion of the first compels the motion of the others, according to a law depending on the nature of the combination. For example, the combination of a crank and connecting rod, with guides and frame, in a steam engine, serving to convert reciprocating into circular motion, would thus be called a *mechanism.*

The term *Elementary Combination* is sometimes used synonymously with *a Mechanism.*

A machine is made up of a series or train of mechanisms; as in the sewing machine, one mechanism serves to actuate the needle, another to move the shuttle, while still another gives motion to the cloth being stitched. All of these mechanisms are actuated by the same source of power, and are so arranged that the motions occur at the proper times.

3. Motion and Rest are necessarily relative terms within the limits of our knowledge. We may conceive a body as fixed in space, but we cannot know that there is one so fixed. If two bodies, both moving in space, remain in the same relative position in regard to each other, they are said to be at rest, one relatively to the other; if they do not, either may be said to be in motion relatively to the other.

Motion may thus be either relative or absolute, provided we assume some point as fixed. In what follows, the earth will be assumed to be at rest, and all motions referred to it will be considered as absolute.

4. Path.—A point moving in space describes a line called its path, which may be rectilinear, or curvilinear. The motion of a body may be determined by the paths of one or more of its points selected at pleasure.

5. Direction.—In a given path, a point can move in either of two directions only, which may be designated in various ways: as up, +, or down, —; to the right, +, or left, —; with the clock, +, or the reverse, —; direction, as well as motion, being relative.

6. Continuous Motion. When a point goes on moving indefinitely in the same direction, its motion is said to be continuous. In this case, the path must return on itself, as a circle or other closed curve. A wheel turning on its bearings affords an example of this motion.

7. Reciprocating Motion.—When a point travels the same path alternately in opposite directions, its motion is said to be reciprocating; it being understood that no part of the path is successively pased over more than once in the same direction.

8. Vibration, a term applied to reciprocating circular motion, as that of a pendulum.

9. Intermittent Motion.—When a piece having motion in alternate directions has definite periods at rest, it is said to have intermittent motion.

10. Velocity.—The motion of a point may be referred to a

point in its path, or to one out of its path In the first case it has *linear velocity*, and in the second, *angular velocity*.

Velocity is either *uniform*, equal spaces being traversed in equal times, or *variable*, unequal spaces being traversed in equal times; increase in the spaces giving *accelerated*, and decrease giving *retarded*, motion.

The uniform velocity of a point is measured by the number of units of linear space passed over in a unit of time. The unit of space is usually one foot, and the unit of time is one second.

According to the above rule, the relation between space, time, and velocity might be expressed by the equation,

$$\text{Velocity} = \frac{\text{Space}}{\text{Time}}$$

$$\therefore \text{Space} = \text{Time} \times \text{Velocity}.$$

Space, time, and velocity are expressed by abstract numbers by comparing them with the units.

A body having a variable velocity is acted upon by a continuous force which accelerates or retards its motion. To measure its velocity, we may suppose the force to cease at a certain point in the body's path; then, the velocity being uniform, it can be measured as above.

In other words, velocity, when uniform, is measured by the space described in a unit of time; when variable, it is measured by the space which would be described in a unit of time, if the point retained throughout that unit the velocity which it had at the instant considered.

11. ANGULAR VELOCITY.—The uniform angular velocity of a point is measured by the number of units of angular space swept over in a unit of time by a line joining the given point with the point outside of its path, about which the angular velocity is desired.

The angular space is here measured by circular measure, or the ratio of the arc to the radius. The unit angle is one subtended by an arc equal to the radius. The number of revolutions in a unit of time also expresses the angular velocity, as one revolution can be represented by 2π in circular measure.

Inasmuch as in circular motion the linear velocity of the point is the velocity along the arc, we may write:—

$$\text{Angular velocity} = \frac{\text{Linear velocity}}{\text{Radius}}; \quad . \quad . \quad . \quad (a)$$

from which

$$\text{Linear velocity} = \text{Angular velocity} \times \text{Radius} \quad . \quad . \quad (b)$$

Thus when the angular velocity remains the same, the linear velocity is directly proportional to the radius.

Example 1.—A wheel 4 feet in diameter turns uniformly on its axis 30 times per minute; what is the linear velocity in feet per minute of a point in its circumference?

Here the angular velocity per minute is

$$30 \times 2\pi = 60\pi;$$

from (b)

$$\text{Linear velocity} = 60 \times 3.1416 \times 2 = 376.99.$$

Answer. 376.99 feet per minute.

Example 2.—How far from the centre of the above wheel will a point lie which is moving at the rate of one mile per hour?

Here the angular velocity per hour is

$$30 \times 2\pi \times 60 = 3600\pi;$$

from (b)

$$5280 = 3600\pi \times \text{Radius}.$$

$$\text{Radius} = \frac{5280}{3600 \times 3.1416} = .467 \ . \ . \ .$$

Answer. .467 feet = 5.6 inches.

The angular velocity when variable can be measured by the same means as that described for variable motion.

12. Revolution and Rotation.—A point is said to revolve about an axis, when it describes a circle of which the centre is in, and the plane is perpendicular to, that axis. When all the points of a body thus revolve with the same angular velocity without changing their relative positions, the body is said to revolve about the axis.

If the axis passes through the body, as in the case of a wheel, the word rotation is used synonymously with revolution.

It frequently occurs that a body not only rotates about an axis passing through itself, but also moves in an orbit about another axis. In order to make the distinction between the two motions more clear, we shall consider the first as a rotation, and the second as a revolution; just as we say, the earth rotates on its axis and revolves around the sun.

13. An Axis of Rotation is a line whose direction is not changed by the rotation; a *fixed axis* is one whose position, as well as its direction, remains unchanged.

A Plane of Rotation is a plane perpendicular to the axis of rotation.

Right-handed Rotation is the same in direction as the motion of the hands of a watch, and is generally considered to be positive. *Left-handed Rotation* is in the opposite direction and is consequently considered as negative.

14. Cycle of Motions.—When a mechanism is set in motion and its parts go through a series of movements which are repeated over and over, the relations between, and order of, the different divisions of the series being the same for each repetition, we have in one of these series what is called a *cycle of motions*. For example, one revolution of the crank of a steam engine causes a series of different positions of the piston rod, and this series of positions is repeated over and over for each revolution of the crank.

The Period of a motion is the interval of time elapsing between two successive passages of a point through the same position in the same direction.

15. Driver and Follower.—That piece of a mechanism which is supposed to cause motion is called the driver, and the one whose motion is effected is called the follower.

16. Frame.—The frame of a machine is a structure that supports the moving parts and regulates the path, or kind of motion, of many of them directly. In discussing the motions of the moving parts, it is convenient to refer them to the frame, even though it may have, as in the locomotive, a motion of its own.

17. Modes of Transmission.—If we leave out of account the action of natural forces of attraction and repulsion, as magnetism, one piece cannot move another, unless the two are in con-

tact or are connected to each other by some intervening body that is capable of communicating the motion of the one to the other. In the latter case, the motion of the connector is of no consequence as the action of the combination as a whole depends upon the relative motion of the connected pieces. Thus motion can be transmitted from driver to follower:

1°. By direct contact.
2°. By intermediate connectors.

18. Links and Bands.—An intermediate connector can be rigid or flexible. When rigid, it is called a link, and it can either push or pull, such as the connecting rod of the steam engine. Pivots or other joints are necessary to connect the link to the driver and follower.

If the connector is flexible, it is called a band, which is supposed to be inextensible, and only capable of transmitting a pull. A fluid confined in a suitable receptacle may also serve as a connector, as in the hydraulic press. Here we might call the fluid a pressure-organ in distinction to the band, which is a tension-organ.

19. Modification of Motion.—In the action of a mechanism the motion of the follower may differ from that of the driver in kind, in velocity, in direction, or in all three. As the paths of motion of the driver and follower depend upon the connections with the frame of the machine, the change of motion in kind is fixed, and it only remains for us to determine the relations of direction and velocity throughout the motion. Now the laws governing the changes in direction and velocity can be determined by comparing the movements of the two pieces in each instant of their action, and the mode of action will fix the law. Therefore, whatever the nature of the combination, if we can determine throughout, the motion of the driver and follower, the velocity ratio, and directional relation, the analysis will be complete.

Either the velocity ratio or the directional relation may vary, or remain the same throughout the action of the two pieces.

20. Graphic Representation of Motion.—We can represent the motion of a point in any given piece of mechanism, graphically, by a right line whose length in units indicates the velocity,

and whose direction indicates the direction of motion of the point at the instant considered; an arrow head is used to indicate the direction in which the point is moving. If the path of the moving point be a curve of any kind, the direction of the curve at any point is that of its tangent at that point, which indicates the direction of the motion as well.

21. RESULTANT.—If a material point receives a single impulse in any direction, it will move in that direction with a certain velocity. If it receives at the same instant two impulses in different directions, it will obey both, and move in an intermediate direction with a velocity different from that of either impulse alone. The position of the point at the end of the instant is the same as it would have been had the motions, due to the impulses, occured in successive instants. This would also be true for more than two motions. The motion which occurs as a consequence of two or more impulses is called the ***Resultant***, and the separate motions, which the impulses acting singly would have caused, are called the *Components*.

22. PARALLELOGRAM OF MOTION.—Suppose the point A (FIG. 1. PLATE I) to have simultaneously the two component motions represented in magnitude and direction by AB and AC. Then the resultant is AD, the diagonal of the parallelogram of which the component motions AB and AC are the sides.

To calculate AD; AC, AB, and the angle CAB being known; we have from the triangle ABD,

$$\overline{AD}^2 = \overline{AB}^2 + \overline{BD}^2 - 2\,AB\,.\,BD \cos ABD.$$

Substituting for ABD the supplementary angle CAB, and extracting the square root,

$$AD = \sqrt{(\overline{AB}^2 + BD^2 + 2\,AB\,.\,BD \cos CAB)}.$$

Conversely, AD may be resolved into two components, one along AB, and the other along AC, by drawing the parallelogram $ABDC$, of which it will be the diagonal.

It very often happens that we wish to resolve a motion into two components, one of which is perpendicular, and the other parallel, to a given line EF (FIG. 2 PLATE 1). Here AD represents the motion; $AB = AD \cos DAB$, the component parallel to EF; and $AC = AD \sin DAB$, the component perpendicular to EF.

23. Parallelopiped of Motions.—If the three component motions AB, AC, and AD, are combined, their resultant AF, Fig. 3 Plate 1, will be the diagonal of the parallelopiped of which these are the edges. The motions AB and AC, being in the same plane, can be combined to form the resultant AE; in the same way AE and AD may be combined, giving the resultant AF.

Conversely, the motion AF may be resolved in to AB, AC, and AD. If the parallelopiped is rectangular, the case is more simple and often used; then we have

$$\overline{AF}^2 = \overline{AE}^2 + \overline{AD}^2 = \overline{AB}^2 + \overline{AC}^2 + \overline{AD}^2.$$

To find the resultant of any number of motions; first combine any two of them and find their resultant; then combine this resultant with the third, thus obtaining a new resultant, which can be combined with the fourth; and so on.

24. Motions of Rigidly-Connected Points.—If two points are so connected that their distance apart is invariable, the component of their motions along the straight line which traverses them must be equal; for if these component motions are unequal, the distance between the points must change. The straight line which traverses the points is called the *line of connection.*

For example, let A and B, Fig. 4 Plate 1, represent a pair of rigidly-connected points, having the motions represented by Aa and Bb; then the components, Ac and Bd of their motions along the line AB must be equal; the components ac and bd perpendicular to AB, having no influence on the motion along AB. In the figure the motions are shown in one plane, but the proposition is also true for motions not in one plane.

If the points A and B are located on a rigid piece, it will have, in consequence of the motions Aa and Bb, lying in one plane a translation along the line AB, and a rotation about an axis.

25. Instantaneous Axis.—If a line OA Fig. 4, is drawn perpendicular to the motion Aa, of the point A, then motion may be the result of a revolution about an axis through any point in the line AO or in AO produced. Similarly, the motion Bb may be

the result of a revolution about an axis through any point in BO. If A and B are rigidly connected, the piece on which they are situated must have a rotation about one axis, and the angular velocity of all points about that axis must be the same. The only point satisfying this condition is O, at the intersection of AO and BO, and the piece has a motion at that instant such as it would have if it were rotating about an axis through O. The axis through O perpendicular to the plane of the motions, is called the *instantaneous axis*, it being the axis about which the body is rotating for the instant in question.

The angular velocity about the instantaneous axis being the same, for the instant, for the points A and B, the linear velocities of A and B will be proportional to their distances from the instantaneous axis;

$$\therefore\ Aa : Bb = OA : OB$$

As in the preceding case, the components along the line of connection must be equal.

If the motions of the points A and B are not in the same plane, the instantaneous axis would be found as follows:

Pass a plane through the point A perpendicular to Aa; the motion Aa might then be the result of a revolution of A about any axis in that plane. In the same manner, the motion of Bb might be the result of a revolution of B about any axis in the perpendicular plane through B. The points A and B, being rigidly connected, must rotate about one axis which in this case will be the intersection of the two perpendicular planes.

Suppose the motions of the two points A and B to be in the same plane and parallel. (FIGS. 5 and 6, PLATE 1). Here the perpendiculars through A and B coincide, and the above method fails. To find the instantaneous axis in these cases, draw a right line through the points a and b in each case, and note the point O where it intersects AB, or AB produced. This must be the instantaneous axis, for by the similar triangles AaO and BbO, we have

$$Aa : Bb = OA : OB,$$

the same condition as was obtained before.

26. INSTANTANEOUS AXIS OF ROLLING BODIES.—The instantaneous axis of a rigid body which rolls without slipping upon the surface of another rigid body must pass through all the points in which the two bodies touch each other; for the points in the rolling body which touch the fixed body at any given instant must be at rest for that instant, and must, therefore, be in the instantaneous axis. As the instantaneous axis is a straight line, it follows that rolling surfaces which touch each other in more than one point must have all their points of contact in the same straight line in order that no slipping may occur between them. This property is possessed by plane, cylindrical and conical surfaces only; the terms cylindrical and conical being used in a general sense, the bases of the cylinders and cones having any figure as well as circles.

Let FIG. 7 PLATE 1 represent a section of the rolling surfaces by a plane perpendicular to their straight line of contact, and assume *PP* as fixed; then *O* is a point in the instantaneous axis, as it is for the instant at rest, and all points on *C*, as *A* and *B*, are rotating about it for the instant. To find the direction of motion of any point, as *A*, draw *AO*, and through *A* perpendicular to *AO* draw *Aa*, which is the direction of motion of *A* for the instant. The linear velocities of *A* and *B* are proportional to their distances from *O*, the instantaneous axis.

27. MOTION OF TRANSLATION.—If, in FIG. 6, PLATE 1, the two parallel motions *Aa* and *Bb* become equal to each other, then *OB* will be infinite, and the consecutive positions of *AB* will be parallel to each other. This is also true if the motions are at any angle with *AB*, so long as they are equal and parallel, as in FIG. 8 PLATE 1.

The motion of a line, or of a body containing that line, at any instant when it is thus revolving about an axis at an infinite distance, is called *translation*. All points in such a body move in the same direction with the same velocity; the paths of the points may be rectilinear or curvilinear. Straight or rectilinear translation is commonly called *sliding*. As an example of straight translation, we have the cross-head of the steam-engine; of curvilinear, the parallel-rod of a locomotive.

28. CENTROID.—The curve passing through the successive positions of the instantaneous centre of a body having a combined motion of rotation and translation is called a *centroid*. The surface formed by the successive positions of the instantaneous axis is called an *axoid*.

29. PAIRS OF ELEMENTS.—In order that a moving body, as *A* (FIG. 9 PLATE 1) may remain continually in contact with another body *B*, and at the same time move in a definite path, *B* would have a shape, which could be found by allowing *A* to occupy a series of consecutive positions relative to *B*, and drawing the envelope of all these positions. Thus, if *A* were a parallelopiped, the figure of *B* would be that of a curved channel. Therefore, in order to compel a body to move in a definite path, it must be paired with another, the shape of which is determined by the nature of the relative motion of the two bodies.

A machine consists of elements which are thus connected in pairs, the stationary element preventing every motion of the movable one except the single one desired.

30. CLOSED PAIR.—If one element not only forms the envelope of the other, but incloses it, the forms of the elements being geometrically identical, the one being solid or full, and the other being hollow or open, we have what may be called a closed pair. The pair represented in Fig. 9 is not closed, as the elementary bodies *A* and *B* do not enclose each other in the above sense.

On the surfaces of two bodies forming a closed pair we may imagine coincident lines to be drawn, one on each surface; and if we suppose these lines to be such in form as will allow them to move along each other, that is, allow a certain motion of the two bodies paired, we will find that only three forms can exist:

1°. A straight line, which allows straight translation.

2°. Among plane curves, or curves of two dimensions, a circle, which allows rotation.

3°. Among curves of three dimensions, the helix, which allow a combination of rotation and straight translation.

31. PRIMARY AND SECONDARY PIECES.—In order to distinguish between pieces of a machine which are connected directly to the

frame and those carried by other moving pieces, the former are called *primary*, and the latter *secondary pieces.*

Thus, if the connection of the primary pieces to the frame be by closed pairs of elements, the following determinate motions can be given to them:

1°. Straight translation or sliding;

2°. Rotation, motion in a circle, as a wheel on its axis;

3°. A helical motion, which might be considered as a combination of 1° and 2°, as a screw.

32. BEARINGS are the surfaces of contact between the frame and the primary pieces, the name being applied to the surface of each piece; but these surfaces sometimes have distinctive names of their own.

The bearings of the primary pieces may be arranged, according to the motions they will allow, in three classes:

1°. For straight translation the bearings must have plane or cylindrical surfaces, cylindrical being understood in its most general sense. The surfaces of the moving pieces are called slides, those of the fixed pieces, slides or guides.

2°. For rotation, or turning, they must have surfaces of revolution, as circular cylinders, cones, conoids, or flat disks. The surface of the solid or full piece is called a journal, neck, boss, spindle, or pivot; that of the hollow or open piece, plumber or pillow block, bush, or step, journal gudgeon or pedestal.

3°. For translation and rotation combined, or helical motion, they must have helical or screw shape. Here the full piece is called a screw, and the open piece a nut.

33. SCREW AND NUT.—A screw might be defined as a solid cylindrical body with a thread or projection of uniform section wound around it in successive equidistant coils or helices; a nut would be formed by winding the thread on the inside of a hollow cylinder. Either the screw or the nut may be the moving piece, the nut being the envelope of the screw in all cases.

The form of a section of the screw thread varies with the use to which the screw is to be put; FIG. 10 PLATE 1 shows some of the common forms.

The most common form is the one shown at *A*, and is known as the **V** thread, its section being an equilateral triangle. As the sharp edges make the thread liable to injury, and less easy to construct, the modified forms *B* and *C* are much used.

Form *B*, known as the Sellers' or the United States Standard, has the angle of the thread 60°, and one-eighth of the depth of the **V** cut off at the top and at the bottom; this makes a better screw, as more material is left between the bottoms of the threads, the very thin parts removed being of little use as bearing surfaces on account of their weakness. Form *C*, known as the Whitworth, or English standard, has the angle of the thread 55°, and one-sixth of the depth is rounded off at the top and at the bottom of the thread.

As the resistance of pieces in sliding contact is normal to the bearing surfaces, there is a tendency in all **V**-shaped threads to burst the nut.

D shows the square-threaded screw, most commonly used to produce motion, as it has large wearing surfaces perpendicular to the motion given by the screw; it is, however, not as strong as *A*, as it has only one-half the shearing surface of *A* in a given length of the nut. *E* is a combination of *B* and *D*, used for screw gearing and the lead screws of engine lathes; the angle being such as will allow a clasp nut to be used, which is not possible with *D*. A modified form of *D*, used for rough work, the screws being cast, is shown at *F*. In *G*, which is used where the force is always applied in the same direction, as in the breech-screws of large guns, the strength of *A* is combined with the flat surface of *D*. Lag screws, used in wood, have the form of thread shown at *H*, here the wood is the weaker material and has the larger thread.

34. RIGHT AND LEFT-HANDED SCREWS.—A screw is said to be right-handed or left-handed, according as a right-handed or left-handed rotation is required to make it advance; and this is a permanent distinction. In FIG. 11, PLATE 1, *R* shows a right-handed, and *L* a left-handed screw; it will be noticed in *R* that

when a right-handed screw is held vertically, the threads will rise from the left to the right on the visible side; in the left-handed screw the reverse is the case.

35. SCREW PITCH.—The pitch of a screw is the distance it advances for one complete turn, and, in single threaded screws, is measured by the distance between two similar parts on successive threads measured on a line parallel to the axis of the screw. Screws are commonly designated by the number of threads to the inch of length; that is, a screw of $\frac{1}{10}''$ pitch, or ten threads to the inch, is called a screw of ten threads per inch.

36. MULTIPLE-THREADED SCREWS.—If instead of winding one thread around a cylinder, several equidistant threads are wound at the same time, taking care in the winding that the threads are kept the same distance apart, we will have a multiple-threaded screw. If two threads are used, a double, if three threads, a triple threaded screw will result, and so on. By the above principle, the pitch can be greatly increased without necessarily increasing the size of the thread. Here the pitch is measured by the axial distance between two similar points on successive coils of the same thread, one point being found from the other by following the thread for one complete turn. The divided pitch is the axial distance between two similar points on successive threads, and in a double-threaded screw is one-half the pitch; in a triple-threaded screw, one-third the pitch, etc.

37. LINEAR SCREW, OR HELIX.—A linear screw, or helix, is a line traced on the surface of a cylinder by a point which has a uniform revolution around the axis of the cylinder, accompanied with a uniform motion along that axis. This line makes a constant angle with the elements of the cylinder, and the distance between the successive coils is therefore constant. A string stretched between two points on a cylinder will in general lie on a helix, as the helix is the shortest line that can be drawn on the surface of a cylinder between two points situated on that surface. In FIG. 12, PLATE 1, *A* shows an elevation, and *B* a development of two coils of a helix; the method of construction is indicated by dotted lines.

38. AXIAL PITCH.—The axial pitch of a helix is the distance from one coil to the next, measured in a line parallel to the axis,

that is, along an element of the cylinder, as $CD=cd$, FIG. 12. PLATE 1. This corresponds to the "screw pitch".

39. CIRCUMFERENTIAL PITCH.—The circumferential pitch is the distance from one coil of a helix to the next, measured along a line at right angles to the elements of the cylinder, and is equal to the circumference of the base of the cylinder, as *Cc*.

40. NORMAL PITCH.—The normal pitch is the shortest distance between two successive coils of the helix, and is measured by the "normal helix" or a line drawn perpendicular to the successive coils, as *cf*. In the discussion of screw gearing, the normal and circumferential pitches, being necessarily referred to some particular cylinder, will be found useful.

The length of one coil of the helix is

$$Cd=\sqrt{(\text{circumference}^2+\text{pitch}^2)}$$

The line of each angle of a screw thread is a linear helix, and all points in or rigidly attached to a turning screw, not lying in its axis, describe linear helices of the same axial pitch; those equidistant, describe equal helices on the surface of the same cylinder; those at unequal distances, describe unequal helices, situated on different cylinders and inclined at different angles to the axis. The angle made by the helix with a line at right angles to the elements of the cylinder is called *the pitch angle*. ✓

41. It will be interesting to note the relation that the slide and journal bear to the screw, from which they might be considered as derived. If we suppose the pitch of a screw to be diminished until it becomes zero, or if we suppose the pitch angle to become zero, then the form *A* (FIG. 13, PLATE 1) would be changed to that of *B*, which, with a modification of the thread outline, would become, like *C*, a common form for a journal. Thus, by making the pitch zero, the action along the axis of the screw has been suppressed, and only rotation is possible for the nut. If we suppose the pitch angle to increase instead of diminish, the screw will become steeper and steeper. If the angle = 90°, the screw threads become parallel to the axis, the screw becomes a prism, and the nut a corresponding hollow prism, as FIG. 13, *D*. Here rotation is suppressed, and only sliding along the axis is possible, giving us the slide. If the angle be made > 90°, the

screw changes from a right to a left-handed one, but still remains a screw.

42. It is very often the case that pulleys or wheels are to turn freely on their cylindrical shafts and at the same time have a motion along them ; for this purpose loose rings or collars (FIG. 14 PLATE 1) are used, the loose collars *a* and *b*, prevent the motion of the pulley along the shaft and allow its free action. Sometimes pulleys or couplings must be free to slide along their shafts, but at the same time turn with them ; they must then be changed to a sliding pair. This is often done by fitting to the shaft and pulley or sliding piece, a key *c* (FIG. 15 PLATE 1), parallel to the axis of the shaft. The key may be made fast to either piece, the other having a groove in which it can move freely. The above arrangement is very common, and is called a *feather and groove*, or a *key and key way*.

43. VELOCITY RATIO.—A screw may be used to produce motion in two ways :—

1°. The nut may be fixed, and the screw be made to turn by applying a force at the end of a lever, or on the circumference of a wheel attached to the screw. While the screw advances through a distance equal to the pitch, the point at which the force is applied describes one coil of a helix of equal pitch. If *P* represents the pitch of the screw, and *R* the shortest distance between the point of application of the force and the axis of the screw, called the lever arm, the velocity ratio is

$$\frac{\sqrt{(2\pi R^2 + \overline{P^2})}}{P}$$

2°. The screw may simply rotate, and the nut may have a motion of translation in a straight line without turning. While the screw makes one turn, the nut will move through a distance equal to the pitch, and the point of application will describe a circle of radius *R* ; the velocity ratio is

$$\frac{2\pi R}{P}$$

The latter form for the velocity ratio is, on account of its simplicity, used as an approximation to the first.

Either of the above combinations may be reversed, that is, the nut may be made to turn and the screw made to remain stationary in 1°, or have a straight translation in 2°. This does not change the velocity ratio.

44. COMPOUND OR DIFFERENTIAL SCREWS.—If A (FIG. 16 PLATE 1) is a fixed nut carrying the screw S, and B is a movable nut, also on the screw S, and free to slide along the guides GG, the pitches of the screw in A and B being P and p respectively, p being $<P$; we will have for each turn of the screw in the direction of the arrow an advance of the screw S to the right equal to the pitch P. Meanwhile the nut B has moved relatively to the screw a distance p to the left. The absolute motion of B is then $(P-p)$, the resultant of the motion relatively to S, and the motion of S. The same result would be obtained by supposing the nuts A and B to act in succession. Thus, suppose B fast to the screw and free to turn, then one turn of the screw in A would advance B a distance $+P$ (motion to the right being positive); now suppose the screw fast in A, turn the nut B back one turn to the position it would have had provided it had not rotated, B will then move a distance $-p$. Adding the two motions, we have for the motion of B, $(P-p)$ as before. This principle of successive movements is very often convenient in determining resultant motions.

If both threads, as in the figure, are right-handed, the motion is, for $P>p$, equal to $(P-p)$ and positive; while for $P<p$, it is negative. When the resultant motion is, as above, the difference of two component motions, the screw is called a *differential screw.*

If one of the threads is right-handed, and the other left-handed, the motion of B would be $(P+p)$, its direction depending on the arrangement and rotation of the screw. A right and left-handed screw are often used in combination to bring two pieces, not capable of turning, together, as in the right and left pipe-coupling. Pieces can also be arranged so as to move equal distances in opposite directions, in reference to some point located between them.

A more practical differential screw than Fig. 16 Plate 1 is shown in Fig. 17 Plate 1 where the screw *S*, working in the fixed nut *A*, is made hollow, and forms the nut for the smaller screw *s*, which is fast to the slide *B*, moving on the guides *GG*. The action is the same as in the previous case.

45. In all the previous cases, the force has been applied to rotate the screw or nut, and thus cause a straight translation; a force causing translation might be applied to a screw or nut, which would cause the nut or screw to rotate. This is not possible with ordinary pitches, as the frictional resistance is so great; it is well known, however, that nuts and screws subjected to constant jarring, such as those on railway trucks, are very liable to work loose, and double nuts, one serving as a check for the other, are often used. When the pitch is made very long, the screw can be easily turned by moving the nut along it; in this case the screw is formed by a steep spiral groove running along in a cylindrical piece. The nut fits this cylindrical piece, and has a projecting feather which fits the groove. This principle is used in a small automatic drill, or boring tool, where the spindle which carries the drill has a multiple-threaded screw of rapid pitch, cut about two-thirds of its length. This screw fits into a tubular handle closed at one end and furnished with a nut which fits the screw; by pushing upon the handle, the screw with the drill is made to rotate; a coiled spring placed between the end of the screw and the closed end of the tube returns the screw to its normal position.

46. Screws are correctly cut in a lathe where the cylindrical blank is made to rotate uniformly on its axis, while a tool, having the same contour as the space between the threads, is made to move uniformly on guides in a path parallel to the axis of the screw, an amount equal to the pitch, for each rotation of the blank. The screw is completed by successive cuts, the tool being advanced nearer the axis for each cut until the proper size is obtained. A nut can be cut in the same way by using a tool of the proper shape and moving it away from the axis for successive cuts.

Screws are also cut with solid dies either by hand or power and with proper dies and care good work will result. Nuts are generally threaded by means of "taps" which are made of cylindrical pieces of steel having a screw thread cut upon them of the requisite pitch; grooves and flutes are then made parallel to the axis to furnish cutting edges, and the threads are tapered off at the end to allow the tap to enter the nut.

Screws cut by open dies that are gradually closed in as the screw is being cut are not accurate, as the screw is begun on the outside of the cylinder by the part of the die which must eventually cut the bottom of the thread on a considerable smaller cylinder. Thus, as the angle of the helix is greater, the smaller the cylinder, the pitch remaining the same, the die, at first, traces a groove having a pitch due to the greater angle of the helix at the bottom of the thread. As the die plates are made to approach each other, they tend to bring back this helical groove to the standard pitch; this strains the material of the threads, and finally produces a screw of a pitch different from that of the die plates.

47. WORM AND WHEEL.—A worm and wheel is a combination of a screw and a wheel furnished with teeth set obliquely on its rim, and so shaped as to be capable of engaging with a screw placed tangential to the wheel, (FIG. 18 PLATE 1). The continuous revolution of the screw or worm, will then impart continuous rotation to the wheel, and it will advance through one, two or three teeth upon each revolution of the screw, according as the thread on the screw is single, double, or triple. On account of the great reduction of velocity obtainable by this combination, it is extremely valuable as a means of obtaining mechanical advantage, and is much used in hoisting machinery. It is also useful in making fine angular adjustments, as in gear-cutting machines; when thus used for making adjustments, it is sometimes called a "tangent screw."

48. POWER OF A SCREW.—Since, if we neglect the loss of work by friction or concussion, any mechanical combination must deliver as much work as it receives, we must have the force at the point of application multiplied by the velocity of that point equal to the force at the point of delivery multiplied by its velocity. Thus, if we call the force applied power, and the force delivered a resistance, we will have

$$\frac{\text{Power}}{\text{Resistance}} = \frac{\text{Velocity of point of delivery}}{\text{Velocity of point of application}},$$

or, the ratio of the power to the resistance is the reciprocal of the velocity ratio.

Thus, in the ordinary case of a screw producing a pressure, we have, neglecting friction, the power multiplied by the circumference of the circle described by the point of application of the power equals the resistance multiplied by the pitch,

$$\text{or} \quad P \times 2\pi R = Wd.$$

49. INVERSION OF CLOSED PAIRS.—If, in a closed pair, we exchange the fixed element for the movable one, there is no alteration in the resulting absolute motion; the exchange of the fixedness of an element with its partner is called the inversion of a pair. This has already been noticed in connection with the discussion of the screw, where it made no difference in the resulting motion whether the screw or nut was considered as fixed. In the ordinary bolt we turn the nut, while in the "tap-bolt" the nut is stationary and the bolt is turned. In FIGS. 16 AND 17 PLATE 1, the blocks B slide in open prismatic slots, while in FIG. 15, PLATE 1, the open cylindrical sliding piece, encloses the guide; the resulting motion is the same in each case. In the common wagon-wheel the axle is fixed to the the body of the wagon, while the nut turns on it; in the railway truck the bearing is attached to the truck-frame, and the axle turns in it with the wheel, which is made fast to the axle.

50. INCOMPLETE PAIRS OF ELEMENTS; FORCE-CLOSURE.—Hitherto it has been assumed that the reciprocal restraint of two elements forming a pair was complete, *i. e.*, that each of the two bodies, by the rigidity of the material and the form given to it, restrained the other. In certain cases it is only necessary to prevent forces having a certain definite direction from affecting the pair, and in such cases it is no longer absolutely necessary to make the pair self closed; one element can then be cut away where it is not needed to resist the forces. When a pair of elements is thus incomplete, and the closure is affected by means of a force or forces, we have what is called a force-closed pair of elements.

The bearings for railway axles, the steps for water-wheel shafts, the ways of an iron-planer, railway wheels kept in contact with the rails by the force of gravity, are all examples of force-closed pairs.

PLATE II.

Harmonic Motion—Velocity Ratio—Manner of Derivation—Conversion of Circular into Reciprocating Motion—Crank and Connecting Rod, Eccentric and Strap—Swash Plate and Escapements.

51. A point in space may undergo three primary cases of motion.

1°. The point may move in a straight line. In such a case the direction of its motion remains constant, being that of the line in which it moves.

2°. The point may move in one plane, but may continually change the direction of its motion.

3°. The point may change its direction so as to move in a curved line of any kind; *i. e.*, it may have the motion of both cases, 1° and 2°.

The motion of a point in a circle is the resultant of two independent movements at right angles to each other and simultaneously impressed upon the given point.

Hence, if we can suppress one of the components of the circular motion, the resultant motion will be rectilinear.

52. Before going farther, it might be well to study the following simple case of relative motions, and determine the velocity ratio of points moving in different paths. Suppose the point *P* (FIG. 1 PLATE 2) to move uniformly in a circle *OPD*, and suppose *N* to move along the line *OD* in such a way that it is always perpendicularly under the point *P*; it is then said to be undergoing harmonic motion. What is the velocity ratio existing between *P* and *N* at any instant of the motion?

As *P* is moving in the arc of a circle, its motion at the instant is in the direction of the tangent of the circle at *P*. Suppose *PT* to be that tangent and to represent the motion of *P* in magnitude and direction at that instant. Now, if we resolve the motion of *P* into components, one perpendicular, and the other parallel to *OD*, the point *N* will move in obedience to the component *PR* parallel to *OD*, the component *PS* perpendicular to *OD* having no influence on its motion.

Thus if we call the angle $PCO = \theta$, we have,

$$\frac{\text{Velocity of } N}{\text{Velocity of } P} = \frac{PR}{PT} = \sin\theta. \quad ✓$$

The modern belief is that the point P (FIG. 1 PLATE 2) while describing the arc OP of the circle $OBDE$, may have been the subject of two independent movements, one from O to N in the direction of the diameter OD, and the other from N to P in a perpendicular direction. Thus let OCD, BCE, (FIG. 1 PLATE 2) be two diameters of a given circle at right angles to each other, P any point in the circumference, PN, PM, perpendiculars on OC and BC respectively. Let P describe the circumference with a uniform velocity; then the point N will travel to and fro along OCD with a variable velocity, while at the same time the point M moves at a varying rate up and down BCE.

This motion of the point N is distinguished by a technical name, *i. e.*, Harmonic Motion. When a point P moves uniformly in a circle, the extremity N of the perpendicular PN, let fall from P upon a fixed diameter OD, has a simple harmonic motion. The term harmonic motion has been chosen for designating one component of circular motion because it represents approximately the motion of a particle in the various media, in which waves of sound, heat, and light are propagated.

Thus a point at the end of a leg of a vibrating tuning fork has an approximately harmonic motion. We conclude that circular motion is of a compound character, and is capable of a resolution into its elements. If it be thus resolved, and if one equivalent be suppressed, so that the motion of N is substituted for that of P, we obtain the fundamental case of the conversion of circular motion into straight line motion, and further we regard circular motion as compounded of two simple harmonic motions in lines at right angles to each other. The characteristic equation representing the rectilinear path of a point undergoing harmonic motion may be derived from FIG. 1 PLATE 2.

Let the path $ON = x$, $CP = a$, $PCO = \theta$,

then $ON = OC - CN$,

✓ or we have $x = a - a\cos\theta$, $\therefore$ $x = a(1 - \cos\theta)$

which is in the form to determine definitely the nature of the motion of a given point N, moving as indicated in Fig. 1 Plate 1, and this expression may be taken to represent the resolution of circular into reciprocating motion. We have seen that the ratio of the velocities of P and N is represented by the sine of the angle PCO, *i. e.*, $\sin \theta$, hence the velocity of N passes from zero at O to the velocity of P at B and then decreases to zero at D.

53. The motion of N may be derived from that of P by a simple mechanical arrangement. In (Fig. 2, Plate 2) let P represent a pin rigidly set in a plate which is secured to a shaft centred at C, and which revolves with the shaft. Let EJB be the sliding piece, JE being at right angles to NB, to which we wish to impart the reciprocating rectilinear motion, in the direction BC. Let this sliding piece be slotted so as to permit the free travel of the pin P along JE and let GG be guides, so arranged as to constrain the rectilinear reciprocating motion of the sliding piece in the required direction BC. Of the two equivalents which combine to produce the circular motion of P that which occurs in the direction EJ is rendered inoperative, and the whole of the other equivalent is imparted to the sliding bar; in this way then, the bar moves to and fro as the plate rotates with the shaft, and any point in the sliding piece, reproduces exactly the motion of N.

Conversely, if the rod BN receive from any source a reciprocating rectilinear motion, as from the piston of a steam-engine, it can impart by this means a circular motion to the plate and shaft—which is the ordinary way of obtaining useful work in the rotation of a shaft, by converting the rectilinear reciprocating motion of the piston rod into the circular motion of the shaft.

54. If instead of the slotted sliding piece and disc of Fig. 2 Plate 2, we suppose a rigid bar to connect C and P and another rigid bar to connect P and some point in the line BC, as Q (Fig. 3 Plate 2), and further suppose the rigid bar to rotate about C, and that CP and PQ have a pin and eye connection, one with the other, we have the mechanism known as the *crank and connecting rod* (Fig. 3 Plate 2). CP is the crank, PQ is the connecting rod,

and *QD* is the sliding piece, moving between guides *G*, *G*. A rotation of *CP* about *C* as a centre will convert the circular motion of *P* into a reciprocating rectilinear motion of *QD*, and conversely, the rectilinear reciprocating motion of *QD*, as in the case of the piston rod, can by the connecting rod *PQ* and the crank *CP*, be converted into a circular motion of *P* about *C*, and hence into the rotation of a shaft centred at *C*.

Let $CP = a$; $PQ = b$; $PCQ = \theta$; $PQC = \varphi$,

then $DQ = CD - CQ = a + b - (a \cos \theta + b \cos \varphi)$

If the connecting rod be made infinite, $\varphi = 0$, and $DQ = a(1 - \cos \theta)$, and the point *Q* has harmonic motion.

A crank with connecting rod of infinite length is an imaginary creation, but an equivalent motion can be obtained in many ways.

Suppose as in Fig. 4, Plate 2 we have a vertical sliding piece *QD*, moving between guides *GG*, and it is desired to impart thereunto a reciprocating, rectilinear, vertical motion of 3 inches, the line of direction of said motion to pass through the centre of a rotating shaft *C*. If on this shaft there be securely fastened a disc or plate, with its circumference in contact with the sliding piece *QD*, and its centre coincident with the centre of the shaft *C*, no motion of translation can be imparted to the sliding piece *QD* by the rotation of the disc or plate, as the locus of the successive points of contact between the perimeter of the disc and the sliding piece is a circle, consequently for every position the distance *QC* is constant. If the centre of the disc is not at *C*, the centre of the shaft, but at some other point *P*, Fig. 4, Plate 2, the sliding piece will receive a vertically reciprocating rectilinear motion, equal to twice the distance *CP* between the centre of the shaft and the centre of the disc. If now we make *CP*, 1.5 inches, we have the motion desired, in kind and quantity.

This mechanical contrivance of a disc keyed to a revolving shaft, the centre of the disc not being at the centre of the shaft, is called an *eccentric;* its object being to convert the circular motion of the revolving shaft, to which it is keyed, and with which it revolves, into a reciprocating rectilinear motion.

It is of common use and is the ordinary method of deriving the motion of the slide valve of a steam engine from the shaft. The analysis of the motion due to an eccentric reveals the fact that it is a particular case of the crank and connecting rod.

55 The eccentric circle supplies a ready method of obtaining the motion given by a crank and connecting rod. The length of the crank is equal to the distance between the centre of motion and the centre of the eccentric circle, as *CP* Fig. 4 Plate 2. Here a circular disc with its centre at *P*, revolves about *C* as a centre of motion, and thereby gives a vertical reciprocating motion to the rod *QD*, moving between guides *GG*. Since *PQ* remains constant as the plate revolves, it appears that *Q* moves exactly as if it were actuated by a crank equal to *CP* and a connecting rod equal to *PQ*. The length of the connecting rod is then the radius of the eccentric circle, and the crank is the eccentricity or distance between the centre of motion and the centre of the eccentric circle. This form of motion is of little value in mechanism, owing to the oblique thrust on *QD*.

56. Fig. 5, Plate 2, shows a method used in practice for utilizing the principle of the eccentric circle. In this mechanism the bar *QD* instead of resting directly on the circle, terminates in a half hoop or strap *AB*. As the circle revolves about *C* as a centre, we have the same crank *CP*, but the connecting rod extends beyond the limits of the circle, and any reciprocating piece, attached to *Q*, can by means of guides as in Fig. 4 Plate 2 be made the recipient of a vertical reciprocating motion, in the line *QC*, through *C* the centre of motion. An eccentric circle can then be used to change the circular into reciprocating straight line motion; it is a particular form of crank and connecting rod, and also a simple form of plate cam. The usual form is to allow the hoop or strap to surround the eccentric circle, as then, the point *Q* is acted upon in both directions; otherwise, *Q* must be returned to its lowest position by a spring or weight.

The throw of the eccentric is always twice the eccentricity, or *Q* can receive an upward motion from its lowest position equal to twice *CP*.

57. Fig. 6 Plate 2 represents the usual form of eccentric used on engine shafts, *CCCC* is the cross section of the shaft centred at *C'*, *P* and *P'* are the extreme positions of the centre of the

eccentric circle. *C′P* is the eccentricity and *PP′* is the throw of the eccentric and equals the travel of the guided point.

The circular half pulleys *DD* of iron or steel are bolted together and embrace the shaft.

Two half hoops of metal, *SS* and *TT*, envelope the eccentric and are bolted together as at *E*. This hoop *SSTT* is loose on the eccentric, so that the eccentric and shaft can rotate about *C′*, without communicating a motion of rotation to *SSTT*. Any sliding piece, moving between guides and attached to *AR*, can be made to take a reciprocating straight line motion, whose line of direction shall pass through *C′*.

The crank and connecting rod mechanism necessitates the division of a shaft whenever it is required to place a crank anywhere except at the end of the shaft, as the connecting rod is continually traversing over the centre of the shaft

58. Fig. 7*a*, Plate 2, shows a manner of dividing a shaft *SS′*, and the attachment of two circular discs *A* and *B* to each portion of the shaft as indicated. A pin *P* connects these discs, and the connecting rod *Q* is attached thereunto.

Fig 7*b*, Plate 2, shows the manner of cranking or bending a shaft *SS* at *P*, so as to attach the connecting rod *Q* in such a manner as to permit its motion; the crank-arm is equal to *ab*.

Fig. 7, Plate 2, shows the manner of employing an eccentric, so as to avoid the division of, or the cranking or bending of the shaft; *AB* is the shaft, divided at *FF*, and receiving a motion of rotation from a connecting rod *QQ*, attached to the pin *P*, and a crank-arm equal to *FF*. *C* is the steam cylinder in which the piston has a reciprocating straight line motion due to the action of the steam. This straight line motion is changed into circular motion by the crank and connecting rod mechanism described. An eccentric at *E* furnishes a reciprocating straight line motion to the piece *ED*, which communicates motion to the slide valve in the steam chest *H*. This action of the eccentric is accomplished without either a division, or bending of the shaft. The great value of the eccentric arises from this feature, which enables us to derive the motion which would be given by a crank and connecting rod, without subdividing or bending the shaft.

59. The crank and connecting rod and eccentric, both produce approximate, simple harmonic motion. The obliquity of the connecting rod causes the small deviation, that exists from true harmonic motion. For a uniform rotation of a crank any point in the connected reciprocating piece has then a simple harmonic motion, approximately. ✓

Fig. 8, Plate 2, shows an arrangement of an eccentric disc A, and a sliding piece QRD moving between guides GG, which produces harmonic motion. CP is the crank, and PQ the connecting rod, of infinite length. This mechanism of the eccentric disc is often used for making delicate adjustments, as by reducing the eccentricity CP, and using the proper part of the swing of the disc, a very slight change in the position of the sliding piece RQ will correspond to quite an angular motion of A, this angular motion being governed by a lever attached to A. ✓

60. The apparatus shown in Fig. 9, Plate 2, known as a *swash plate*, consists of a circular plate A set obliquely upon the shaft S, which by its rotation causes a sliding bar CD to move up and down in a line parallel to the axis of the shaft, between the guides GG; the friction between the bar and the plate is lessened by a roller O. When a roller is used the motion of CD is approximately harmonic; the smaller the roller, the closer the approximation. If a point is used in place of the roller, the motion is harmonic.

For (Fig. 10, Plate 2), let eba represent the inclination of the plate to the axis of the shaft, eof the actual path of the point O on the plate, and the dotted circle erd the projection of this path on a plane through e (the lowest position of O) perpendicular to the axis of the shaft ab. Draw om perpendicular to ef, and or perpendicular to the plane erd, and draw rn perpendicular to ed, the diameter of the circle erd. Join mn and suppose the plate to rotate through an angle $ear = \theta$, and thus to carry the point O through a vertical distance or. Then

$$or = mn = ab \times \frac{en}{ea}, \text{ since } \frac{mn}{ab} = \frac{en}{ea} \quad ✓$$

$$mn = ab\left(\frac{ea - an}{ea}\right)$$

$$mn = ab\left(1 - \frac{an}{ea}\right)$$

$$mn = ab(1 - \cos\theta)$$

which is the same expression as was derived in the case of harmonic motion.

In this case *ab* represents the length of the crank and is equal to half the travel of the sliding piece *CD*.

61. FIG. 11, PLATE 2, shows the form of a mechanism that will give an intermittent vertical, reciprocating straight line motion. The motion is given to the piece *SS* through the rotation of the plate *P*, about *C* as a centre, and the consequent action on *FF*. This plate is a circular equilateral triangle formed by three circular arcs, whose centres are at *A*, *B* and *C*, respectively, and is embraced by the sides of the rectangular opening in the disc *FF*. As *CAB* rotates about *C*, the portion *CB* will raise the disc *FF*; the point *B* will then act on *aa* and raise the disc still higher, as the edge *AB* passes under *aa*, there will be no lifting action, as *AB* is the arc of a circle about *C*. The disc will therefore rest during this period and then begin to fall in a manner, the reverse of that, in which it rose.

Suppose the circle described by the point *B*, to be divided into 6 equal parts and numbered as in FIGS. 12 and 13, PLATE 2, and let the line *aa* represent the sliding piece. Then as the point *B* moves from 1 to 2, the frame remains at rest; from 2 to 3, the arc *CB* drives the line *aa*; from 3 to 4 the point *B* drives; from 4 to 5 there is rest; and from 5 to 6 motion downwards; until when the point *B* reaches 1, the line *aa* comes to rest. The driving point *B* is the only one considered in this graphical analysis of this mechanism.

62. The object of the different mechanisms mentioned in this connection is to produce either a simple harmonic action, by changing circular into reciprocating straight line motion, or the reverse.

In all cases except the last the motion is continuous. The last mechanism described has for its object a vertical reciprocating straight line motion with periods of rest, and types a class of mechanisms for producing what is known as *intermittent motion*.

Circular may be converted into reciprocating motion by the aid of *escapements.*

An escapement consists of a wheel fitted with an odd number of teeth which are made to act upon two distinct pieces or pallets, attached to a reciprocating frame, and it is arranged, that when one tooth escapes or ceases to drive a pallet, the other shall commence its action.

63. FIG. 14, PLATE 2, illustrates a simple form of escapement. A sliding frame *A*, moving between guides *GG*, is furnished with two projecting pieces at *C* and *D*, and within this frame is centred a wheel possessing three teeth *P*, *R* and *Q*, which wheel rotates in the direction of the arrow.

The upper tooth is now, by its action on the pallet *D*, driving the frame *AB* to the right; when the tooth *P* escapes from *D*, by the rotation of the wheel on its axis, the tooth *Q* will just be brought into contact with the pallet *C*, and by its action thereon will force the frame *AB* to the left, until *Q* escapes from *C*. *R* will then come in contact with *D*, and the frame *AB* will be again driven to the right. Thus a continuous rotation in one direction of the wheel carrying the teeth *P*, *R* and *Q*, produces a reciprocating straight line motion in the frame *AB*.

It will be seen that the wheel must have an odd number of teeth, if it were otherwise the teeth would be at the opposite ends of a diameter of the wheel and would engage both pallets at once, and no motion of the frame *AB* would be possible.

64. The crown wheel escapement FIG. 15, PLATE 2, was invented for the earliest clock of which we have any record. The form of the wheel is that of a circular band, with an odd number of large saw shaped teeth cut upon one edge; the vibrating axis *AB* carries two flat pieces of steel *a* and *b* called pallets, which project from this axis *AB* in directions, at right angles to each other, and so engage, alternately, with teeth upon the opposite sides of the wheel, whenever rotation is communicated to the wheel. In order that each pallet may be struck in succession, either the wheel must have an odd number of teeth, or the axis that carries the pallets must be set a little out of the central line.

Suppose the wheel to turn towards the right hand as we look at it, and let one of the teeth encounter the pallet *b* and cause it to move out of its path; as soon as *b* escapes, a tooth on the opposite side meets the pallet *a* which is now in an approximately vertical position, and causes it to move to an approximately horizontal position and thus bring the axis *AB* back to its former position; thus a reciprocating action is set up in *AB*; this action being in the nature of a vibration or oscillation about a centre, instead of a sliding as in the frame *AB*, Fig. 14, Plate 2, some counterpoise weight is necessary to control the vibration of *AB*, else it would be very rapid. This is accomplished by the heavy arm *CD*, with weights at the end of the bar. This arm is secured at right angles to *AB* as shown in Fig. 15, Plate 2. The recoil set up in the wheel by the inertia of the arm *CD* materially subtracts from the utility of the contrivance.

This simple form of escapement was used in the first clock ever made and marked the time beats with its step by step movement. It was the foundation idea from which were perfected the anchor, Graham's dead beat, and cylinder escapements now used in clocks and watches. It is worthy of note, that the improvements that removed the defects of the early forms of escapements and perfected their form for use in clocks, separate the combination entirely from its original conception, viz: that of an apparatus for converting circular into reciprocating motion; no such conversion can be effected by the modern clock escapement.

PLATES III, IV AND V.

Cams.

65. Circular may be converted into reciprocating rectilinear motion by the aid of cams.

A CAM is a curved plate or groove which communicates motion to another piece by the action of its curved edge. This motion is usually transmitted by sliding contact; but where there is much force transmitted, the contact is often made rolling.

WIPER.—If the action of the piece is intermittent, it is sometimes called a *wiper;* that is, a cam, in most cases, is continuous in its action, while a wiper is always intermittent; but a wiper is often called a cam notwithstanding.

In most cases which occur in practice, the condition to be fulfilled in designing a cam does not directly involve the velocity ratio, but assigns a certain series of definite positions which the follower is to assume while the driver occupies a corresponding series of definite positions.

The relations between the successive positions of the driver and follower in a cam motion may always be represented by means of a diagram, such as is given in FIG. 1, PLATE 3, where the line *Oabc* represents the motion given by the cam. The perpendicular distance of any point in the line from the axis *OY* represents the angular motion of the driver, while the perpendicular distance of the point from *OX* represents the corresponding movement of the follower, from some point considered as a starting point. Thus the line of motion *Oabc* indicates that from the position 0 to 4 of the driver, the follower had no motion; from the position 4 to 12 of the driver, the follower had a uniform upward motion *b*,12; and from position 12 to 16 of the driver, the follower had a uniform downward motion *b*,12, thus bringing it again to its starting point.

66. GENERAL CASE.—Suppose we have given (FIG. 2, PLATE 3), the position of the centre *C* of a plate cam; that the cam turns

uniformly left-handed on its axis and gives motion to the sliding piece *QD*, in the vertical straight line through *C*, by means of the roller *A*, that the point *D* shall remain stationary for the first ¼ revolution, move uniformly upward an amount *LE* (FIG. 1 PLATE 3) for the next ½ revolution, and then move uniformly downward an amount *LE* for the remaining ¼ revolution ; to find the outline of the cam.

First draw the motion diagram (FIG. 1 PLATE 3), where *Oc* is taken to represent 360° of motion of the cam or driver, and is divided into sixteen parts, giving seventeen points at which ordinates can be erected, showing the corresponding movements of the follower. For the first ¼ revolution, from 0 to 4, there is no motion of *QD*, thus giving the line *Oa* coincident with *OX*; for the next ½ revolution, from 4 to 12, there is a uniform upward motion *b*, 12, equals *LE* ; this would be indicated by the straight line *ab*, and any intermediate position of *D* between 4 and 12 could be found by drawing ordinates at the cam position ; similarly, for the next ¼ revolution *bc* would represent the motion of *D*, relatively to the cam. Draw the ordinates at the points of division of *Oc*, and transfer them to *LE* as indicated, marking the points as indicated. The motion of *QD* might have been, and often is, other than uniform ; in that case, the line *abc* would have been curved ; the ordinates could be used as before. The straight line has been used here, as it gives a clearer figure.

The sliding piece *QD* is shown in its lowest position. Describe a circle about *C* that shall pass through the centre *D* of the roller *A* in its lowest position ; as the point is not to begin its upward motion for ¼ revolution, the cam outline for 90° is evidently, the arc of 90° of this circle just drawn. Divide the circumference of the circle into the same number of equal parts as the line *Oc* in the motion diagram, and through the points of division of this circumference, draw radii, prolonged beyond the circumference, number them to correspond with the number of the ordinates in the motion diagram, 1, 2, 3, etc. About *C* as centre describe arcs of circles, with radii equal to the radius of the circle through the lowest position of the roller, increased successively, by the ordinates to *OC* in the motion diagram, and number these arcs to correspond with the same ordinates, 1, 2, 3, 4, etc., the intersection of each arc with the radius having the same number, is a point of the cam outline. Determine a sufficient number of such points, and join them with a smooth curve, and it is the cam outline required. Or having the ordinates at the points of division of *OC* transferred to *QD*, about *C* as a centre, and through these points of division, 0, 6, 8, 10, 12, etc., describe circles, their intersection with the prolonged radii marking the division of the circumference, and corresponding in notation to these ordinates will be the points of the cam outline desired. This construction determines only the outline of a cam, that would drive the point *D*. If a roller is to be used, the cam outline must be located at a distance from the determined outline through *A* equal to the radius of the roller, and parallel in outline, to the outline determined. A convenient method of determining this parallel cam outline, is to describe a series of small arcs from various points of the determined cam outline, with a radius equal to the radius of the roller. A sufficient number of such arcs are then described from points distributed around the cam outline determined, and in the direction in which the parallel outline is required. A smooth curve, tangent to these arcs, *i. e.*, the envelope of these small arcs, will be the cam outline.

67. Cams are usually supplied with rollers as they greatly reduce the friction and wear. It will be noticed that the positive action of the cam is only to drive the roller away from the centre *C*; if there be much friction, the return to the lowest position must be provided for by a spring or weight. Now if we suppose the cam plate to be extended beyond the roller, and cut a groove in the plate, which groove shall be the envelope of the successive positions of the roller, it will act on the roller equally well in either direction. The groove in this case would have parallel sides and should be a little wider than the roller to prevent binding, the play being allowed by taking material from the non-acting surface of the cam.

When a cam actuates its follower equal well in both directions, without external aid it is called a positive motion cam, the elements being so paired that only one motion is possible between them.

68. There are other means than the use of a groove for insur-

ing the positive motion of the follower not open to the objection of binding the follower roller in its groove.

Two rollers might be used, working on the opposite sides of the same cam, and always situated diametrically opposite each other; in such a case the original cam outline, that is, the outline passing through the centre of the rollers, must have a constant diameter equal to the distance between the centres of the rollers.

Such a cam is shown in FIG. 3, PLATE 3, where the rollers *a* and *b* bear on opposite sides of the cam, and are carried by the frame *cc*.

69. Two cams might be arranged side by side on the same shaft, and act on two connected rollers, one running on one cam and the other on the other cam; this would render a more complicated motion than that shown in FIG. 3, PLATE 3, possible.

Two cams might also be used as in FIG. 4, PLATE 3, turning on separate shafts but uniformly in relation to each other.

Here the cams are counterparts of each other, and have the sums of the radii from the centres of *A* and *B* to the centres of the rollers a constant. This operation has been used to operate the harnesses of looms, by connecting the end *D* of the lever *CD* to the harness frame.

70. It is often the case that only a few positions of the follower of a cam are fixed, and it is not particular what the intermediate motion is. In such a case, the outline of the cam may be constructed by passing arcs of circles through the fixed points of the cam in such a way as to make a smooth curve.

FIG. 5, PLATE 3, shows such a cam as applied to the lever of a punching machine. Here the cam revolves in the direction of the arrow, the left hand end of the lever is raised by the action of the arc of the cam extending from *b* to *c*, thus bringing the punch, the carrier of which is worked by the end of the lever to the right of the fulcrum *B*, down to the metal to be punched. A roller *r* is placed at *c* to lessen the friction, while the work of punching is being done; after the punching, the lever is lowered by the action of the arc *cd*.

The part of the arc *dab* is made circular about the axis *C*, and for nearly half a revolution of the cam the lever remains at rest, with the punch raised from the metal, thus allowing the workman time to adjust the plate for the next stroke of the punch. The cam is often arranged to slide on its shaft by means of a key and keyway, and is brought into position under the lever, when required, by means of a foot treadle, moved by the operator.

71. FIG. 6, PLATE 3, shows the manner of obtaining two reciprocating straight line motions at right angles to each other by the use of a plate cam with two grooves.

The cam is one used in sewing machines to give the vertical reciprocating straight line motion to the needle, and the horizontal reciprocating straight line motion to the shuttle.

The needle carrying bar, *a*, is constrained to move up and down in a vertical line between guides *G. G*, it is driven by the lever *BDP*, which has a centre of motion at *D*, and at the end, *P*, a roller that moves in the groove *nn*, in the cam plate. *C* is the centre of rotation of the cam plate. The eccentricity of the groove *nn* with reference to *C*, the centre of rotation, gives the point *P* its vertical motion which is communicated to the needle, *D* being a fixed centre of motion for *P*. The only motion *P* can undergo is one in an arc of a circle about *D*.

The shuttle moves to and fro in its path *bE*. Motion is communicated to the shuttle by a rod *AA*, which is free to slide between the guides *df*, at the other end of *AA* is a roller *V*, which moves in the groove *mm*. The only motion *AA* can undergo, is a horizontal motion between the guides, and as the groove *mm* is eccentric with reference to *C*, the centre of rotation of the cam plate, the rotation of the cam will cause *AA*, and therefore *S*, to reciprocate, an amount equal to twice this eccentricity.

72. FIG. 7, PLATE 3, illustrates another method of accomplishing the motion of the needle, by means of a heart-shaped piece *SS*, grooved, as in the diagram, and carrying the needle bar *AB*, and a rotating disc *C*, carrying a pin *P* that moves in the groove of the heart-shaped piece *SS*. The action is as follows: The disc rotates, and when *P* gets to the top of the groove, it will raise the heart-shaped piece and the bar *AB* and needle *N*. Further rotation of *C* will continue to raise *AB* while *P* apparently descends in the groove, until the state of affairs shown in the dotted position is reached, *i. e.*, *P* is at the bottom of the

groove and in its own highest position. The bar *AB* is then in its highest position and further rotation of *C* will cause it to descend.

73. Fig. 8, Plate 3, shows a cam used in raising the hammer in the striking mechanism of a large clock. The hammer rises and falls with the lever *AB*.

Fig. 9, Plate 3, shows another form of cam plate. *CD* is a rectangle with a slot *RS* cut through it obliquely ; a pin *P*, fixed to the sliding bar works in the slot *RS*. If the rectangle *CD* be moved in the direction *RS* it will impart no motion to the sliding bar *AB;* but if it be moved in any other direction, the pin *P* will be pushed to the right or left, and a longitudinal movement will be given to *AB*. Conversely if *AB* have a reciprocating motion between the guides *G.G*, it will communicate by means of the pin *P* and the slot *RS*, a reciprocating straight line motion to *CD* at right angles to *AB*.

74. Fig. 10, Plate 3, shows an application of this principle to a bar for cutting the groove in rifled guns. *HH* is the rifling bar, and is made hollow, and the toolholder *AB* in the rifling head *FF* is made to move in and out, with reference to the groove that is being cut, by means of a pin *P'*, working in a slot *RS* in the feed rod *PP;* as the feed rod *PP* is pushed in either direction along the axis of the bar *HH*, the cutter *E* will also move in or out in the direction *A'B'*.

There are two cases that may be considered in this form of cam (see Plate 3):

1. Suppose that *CD*, Fig. 9, is moved at right angles to *AB*. Then in Fig. 11 draw *RN* perpendicular to *AB*, and *RS*, making $RPB = RPB$ in Fig. 9. Then (Fig. 9)

$$\frac{\text{travel of } CD}{\text{travel of } AB} = \frac{RN}{PN} (\text{Fig. } 11) = \tan RPN.$$

2. Suppose *CD*, Fig. 9, moves in a direction inclined at any given angle to *RS*. In Fig. 12 draw *RN* making the given angle with *RS*, and draw *RS* as in Fig. 11, then

$$\frac{\text{travel of } CD}{\text{travel of } AB} = \frac{RN}{NP} = \frac{\sin RPN}{\sin NRP}$$

When the sum of the angles at R and P, FIG. 12, equals one right angle, the velocity ratio of CD to AB is expressed by the tangent of RPN, for in that case $\sin NRP = \cos RPN$, and the velocity ratio of CD to AB equals $\frac{\sin RPN}{\cos RPN} = \tan RPN$.

If we suppose CD, FIG. 9, to be wrapped around a cylinder, as in FIG. 13, it will form a screw thread, and the revolution of the cylinder upon its axis, will give by means of the pin P the same longitudinal motion to the bar AB as described in FIG. 9.

The same motion, then, can be derived from a flat plate, with a groove, and from a helical groove on a cylinder, as seen from FIG'S 9 and 13, PLATE 3.

75. Where the cam plate is required to effect more than one double oscillation of the follower during each revolution of the cam, its edge must be formed into a corresponding number of waves, as in FIG. 14, PLATE 3. This is an illustration of a telegraph commutator, the interruptions of the current being caused by the vibrations of the lever PCQ, centred at C. These vibrations are caused by a pin S, travelling in the groove *nnn;* as the wheel revolves it can impress a number of double oscillations upon the lever PCQ.

[illegible]. The following constructions of cam outlines will indicate the process to be followed in any case.

FIG. 1, PLATE 4. Having given a revolving shaft centred at C, to construct a plate cam for this shaft, that shall give a sliding piece, a vertically reciprocating rectilinear motion of 3 inches, whose line of direction shall pass through the centre of the shaft.

FIG. 1, PLATE 4, shows an eccentric with an eccentricity $CP = 1.5$ inches and consequently a throw QQ' of 3 inches, that will give to the sliding piece QD, moving between the guides $G.G$, the required motion. If it be required that the motion be uniform this construction will not answer, since for equal angular movements of the plate in revolving about C as a centre, there will not be equal vertical movements of QD. Some other construction will therefore be necessary. FIG. 2, upper diagram, shows a cam outline that will accomplish this uniform motion. The lower diagram is the required motion diagram. Take AB in the lower diagram, FIG. 2, to represent 360° of the cam motion of rotation; divide it into any number of equal parts, as 8, and designate them as in the figure. Let Q in the upper diagram, FIG. 2, be the lowest position of the centre of the roller, make Ax in the lower $= QC$ in the upper diagram and draw xz parallel to AB; at the centre division erect the perpendicular yse, making ys equal to 3 inches, join x and y, and y and z; now if the figure $AxyzB$ be cut from some rigid material, and if it be given a reciprocating motion at right angles to, and beneath QD, so that the outline xyz shall pass beneath Q, and the extent of the movement shall equal xz, it will give the required movement, the point Q, thereby, rising uniformly a distance, ys, and falling uniformly the same distance, ys, during the travel of the piece $AxyzB$ beneath it. If now we can transfer this outline to the circumference of the circle about C through the lowest position of Q, the resulting outline will by its revolution about C as a centre accomplish the same result. To do this, divide the circle about C through Q into the same number of equal parts, 8, as the line AB in the motion diagram, through the points of division thus determined, draw the prolonged radii, Ct_2, Ct_3, etc., and mark them as indicated.

The outline xyz is the motion diagram, the line AB is drawn below, parallel and at a distance from $xsz = QC$, the distance from the centre of the shaft to the lowest position of the point to be raised. The proper radius of the cam outline at any point can be at once determined and taken from this diagram; if the motion of the cam during one revolution is divided into 8 parts, this circle through Q is divided into 8 parts, as is the line xz, and prolonged radii are drawn through the points of division of the circle QC, and ordinates erected at the points of division in the motion diagram. To find the point of the cam outline, that shall be under the point Q, when the cam shall have turned 90° left-handed, or ¼ revolution, we take from the motion diagram the distance cc' with dividers, and lay it off from C on the radius Ct_3 and so determine t the corresponding point of the outline of a cam acting on a point only, at Q. This construction is general where the line of direction of the motion passes through the centre of the shaft.

Then, from the motion diagram, take as radii the lines bb', cc', *dd'*, *ee'*, and describe arcs about *C* as a centre ; their intersection, respectively, with the radii, Ct_2, Ct_3, Ct_4, Ct_5, will determine points of the cam outline, *t*, *t*. It will be observed that any radius as *cc'* will determine two points of the cam outline by the intersection of the arc described with it about *C* with the corresponding radii Ct_3 and Ct_7 : a smooth curve joining the points thus determined, will give the outline *t . t . t . t;* parallel to this, and at a distance from it equal to the radius of the roller, construct the outline *t' . t' . t' . t'*, as described ; this is the cam outline required.

77. Another construction is as follows: FIG. 3, PLATE 4. On the line of direction *DCF*, and above *Q*, the lowest position of the point to be moved, lay off *QQ'* equal to 3 inches, the required movement, divide this distance *QQ'* into any number of equal parts, as 8, and number them as indicated, and through these points of division, and about *C* as a centre, describe concentric circles; divide the circle through *Q*, into the same number of equal parts, and through the points of division draw the prolonged radii marked 1, 2, 3, 4, 5, 6, 7, 8, respectively ; the intersection of these radii with the concentric circles of the corresponding notation, are points in the required outline ; a smooth curve, parallel to this determined outline, and at a distance from it, equal to the radius of the roller, is the required outline of the cam.

78. 2. To construct a plate cam, that shall give to a sliding piece, a period of rest for ¼ revolution, a uniform rise of 3 inches in the next ¼ revolution, a uniform fall of 3 inches in the next ¼ revolution, and a period of rest for the next ¼ revolution ; the line of direction of the motion to pass through the centre of the shaft, and the point, in its lowest position, to be 3 inches above the centre of the shaft. The construction is as follows:

FIG. 4 PLATE 4, UPPER FIGURE.—With $QC = 3$ inches describe a circle about *C;* as the point is to rest during the first and last quarters of its revolution, the cam outline for these periods will be the semi-circumference of the circle just drawn, *t t t t*. It

then has for the other half of its revolution a uniform reciprocation of 3 inches. Construct the motion diagram, *x y z s t*, *xy* and *st* being parallel to *AB*, since during the periods corresponding to these distances, there is no movement. Then arcs described with the lines *dd'*, *ee'*, *ff'* as radii about *C* by their intersection with the corresponding prolonged radii C_4, C_5, C_6, determine, as before, the points of a curve from which the cam outline is determined as in case 1. In case any other kind of motion is desired, such as accelerated, or uniformly accelerated or retarded, the motion diagram would show a curved outline in place of the broken line *xyz*, in each instance.

FIG. 7, PLATE 4, indicates the method of constructing the curve of harmonic motion, which can be applied with advantage to cam outlines. An inspection of FIG. 7*b* reveals the fact, that if this curve be used for that part of the cam outline that accomplishes the upward movement of the sliding piece, and if the cam outline that accomplishes the downward movement be symmetrical with the other half of the cam, that the change from an upward to a downward motion will be gradual, and hurtful shocks will thereby be avoided in rapid moving or delicate mechanisms actuated by cams. These examples indicate the method to be employed in all cases of constructing the cam outline, when the line of direction of the required motion passes through the centre of the cam. In the cases cited, the circumstances of motion are the same, whether the cam revolves right-handed or left-handed.

79. In the following cases the cams both are supposed to turn left-handed, and the line of direction of the motion is vertical and 3 inches to the right of the centre of the shaft.

3. FIG. 5, PLATE 4.—Required the outline of a plate cam, to give to a sliding piece a vertically uniform reciprocating rectilinear motion of 3 inches, the line of direction of the motion to pass 3 inches to the right of the centre of the shaft, and the point in its lowest position to be 3 inches above the horizontal line through the centre of the shaft.

Lay off $CH = 3$ inches, and *HQ*, at right angles to $CH = 3$ inches, *Q* is the lowest position of the point. Through *H* draw the circle indicated about *C* as a centre. Then any upward motion of *Q* can only be accomplished by an increase of the length of the line *QH*, as the cam plate revolves; the line *QH* in any position will be tangent to the circle through *H* about *C*, hence if, at equidistant points of the circumference of this circle *CH*, we construct tangents and make them equal to *QH* plus the proper proportional part of the required vertical movement, the extremities of the tangents will be points of the required cam outline, which points being joined by a smooth curve, we have the cam outline required; the construction is as follows:

Take in the motion diagram, *xz* to represent the motion of the cam through 360°, lay off $Ax = QH$ and draw *AB* parallel to *xz*, divide *AB* into 8 equal parts and erect ordinates, make $sy = 3$ inches, connect *x* and *y*, and *y* and *z*, and designate the ordinates as indicated. Next divide the circle about *C*, through *H*, into 8 equal parts and mark the points of division from 1 to 8, as in the figure; construct tangents at each point of division, as indicated, and mark them; then lay off on these tangents, from the point of tangency as shown, the distances bb', cc', dd', respectively, and thus determine the tangents $2t$, $3t$, $4t$, etc., in extent; the extremities of these tangents are in the cam outline *t t t t*. By drawing a circle about *C* through *Q* and constructing the tangents exactly as before, a means of using the increments to *QH* is obtained. These increments being the ordinates above *xz* in the motion diagram, taken successively.

80. 4. FIG. 6, PLATE 4.—Required the outline of a plate cam to turn left-handed, and in the first ½ revolution, to raise a sliding piece uniformly 3 inches, to permit it to drop suddenly to its first position, and to permit a period of rest for the remaining ½ revolution; the line of direction to be vertical, and 3 inches to the right of the centre of the shaft that carries the cam, and the lowest position of the bearing point to be 4 inches above the horizontal through the centre of the shaft *C*.

First construct the motion diagram as before, *xy* corresponding to the uniform upward movement, *ys* to the drop to the first position and *sz* to the period of rest. Next lay off $CH = 3$ inches, $HQ = 4$ inches. Draw the circles about *C* with *CH* and *CQ* as radii, respectively. As the last ½ revolution of the cam plate produces no motion in the sliding piece, its outline for the last 180° must be the semi-circumference $QQ'Q''$ through Q, about C, and to the left, as the cam turns left-handed. The tangent $t_5 5$ to the circle about C through H will mark the drop. To determine the points $t\,t\,t\,t$ lay off in the motion diagram $Ax = QH = 4$ inches, and make the divisions and notation as before, and construct the tangents at the derived points and make their lengths $= bb'$, cc', dd', and ey, respectively, the line $5t$ for example $= ey$. The curve drawn through these points, $t\,t\,t\,Q''Q$, is the cam outline required.

81. In the same manner as suggested for cases 1 and 2, any desired curve can be employed in the motion diagram, and the desired radii of the required cam outline be determined in the same manner as herein outlined. These indicated solutions will suggest the methods to be employed in constructing the outline for a plate cam to give any required simple movement in one plane. Simple movements may be combined and a resultant movement obtained by cams.

82. All cams thus far discussed have completed their action in one revolution; by suitably shaping the follower, this action can be extended so as to require more than one revolution, but the cams then become complicated and are not much used on that account.

FIG. 1, PLATE 5, shows a cam where two revolutions are necessary to complete its action. The follower has the elongated form shown at *F*, in order that it may properly pass the intersection of the cam groove, which is the envelope of the follower. If the cam revolves right-handed, there is a period of rest for the follower during the first ½ revolution of the cam, due to the circular groove *ab*, during the second ½ revolution the lever moves to the dotted position under the action of *bc*; *cd* then retains it for the third ½ revolution, and *da* returns it to its first position during the remaining ½ revolution.

83. CYLINDRICAL CAMS.—A cam may be made by cutting the proper groove around a cylinder; the motion of the follower would then be in a direction parallel to the axis of the cam. Such a cam is shown in FIG. 2, PLATE 5, *A* produces a motion similar to that produced by the cam shown at *B*. Both forms are very extensively used to actuate the feed mechanisms used

in machine tools; their period of action only occupies a small part of their revolution, and comes just before the tool is ready for a cut.

A cam like that shown in FIG. 2 can be constructed by the laying out, upon paper, its motion diagram (FIG. 2, *C*) on a line *ab*, equal in length, to the circumference of the cam cylinder, and wrapping this diagram around the cylinder, taking care that the line *ab* is kept in a plane perpendicular to the axis of the cylinder. The centre line of the groove being thus determined, the groove can be made the envelope of the follower when it moves along this centre line. The drawing of the cam can be made from its diagram or development by the same method that is used in drawing the helix.

A groove having a centre line *acdeb* (FIG. 2, *C*) might be cut in a flat plate which has a rectilinear motion along *ab;* a follower moving in a line perpendicular to *ab*, it being supplied with a roller working in the groove, would have a motion very nearly the same as that obtained by *A* and *B*. If the lines of motion in *A* and *B* were straight instead of curved the roller in *C* would have exactly the same motion as in *A* and *B*. Cam grooves cut in plates having a rectilinear motion, are often used in practice.

84. A cylindrical cam having a helical groove is shown in FIG. 3, PLATE 5. The follower, *F*, is made to fit the groove sidewise, and is arranged to turn on the sliding rod, *F*, to which it gives motion in a line parallel with the axis of the cam. The guides for this cam are attached to the bearings of the cam, *A* and *B*, which form a part of the frame of the machine. A plan of the follower is shown at *G:* its elongated shape is necessary, so that it may properly cross the junctions of the groove. In this cam there is a period of rest during ½ a revolution of the cam at each end of the motion, the motion from one limit to the other is uniform, and consumes one and one-half uniform revolutions of the cam.

The cylinder (FIG. 3) may be increased in length, and the groove may be made of any desirable pitch; the period of rest can be reduced to zero, or increased to nearly one revolution of the cam. A cylindrical cam, having a right- and a left-handed groove is often used to produce a uniform reciprocating motion, the right- and left-handed threads or grooves passing into each other at the ends of the motion, so that there is no period of rest.

85. The period of rest in a cylindrical cam, like that shown in FIG. 3, PLATE 5, can be prolonged through nearly two revolutions of the cylinder by means of the device shown in FIG. 4, PLATE 5. At the junction of the right- and left-handed grooves with the circular groove a switch *a* is placed, and it is provided that the switch shall be capable of twisting a little in either direction upon its supporting pin, while the pin is capable of a slight longitudinal movement parallel with the axis of the cylinder. This supporting pin is constantly urged to the right by a spring, shown in FIG. *A*, which acts on a slide carrying the supporting pin; when in this position the space between the switch and the circular part of the groove is too small to allow the follower to pass, and when the follower is in the position shown in *B*, the spring is compressed; then, if the follower moves on, the space behind it is closed, as the spring will tend to push the supporter to the right, and swing the switch on the follower as a fulcrum.

If the cam revolves in the direction of the arrow, in *A*, the shuttle-shaped follower is entering the circular portion of the groove, and leaves the switch in a position which will guide the follower into the circular groove when it again reaches the switch; in *B*, the switch is pressed toward the left to allow the follower to pass. As motion continues, the support of the switch is pressed to the right, and the switch is thrown into the position shown in *C* ready to guide the shuttle into the returning groove. The period of rest in this case continues for about one and two-third revolutions of the cylinder.

If the path of the follower, in FIG. 3, PLATE 5, is inclined to the axis of the cam, the groove would be cut in a cone, or hyperboloid, generated by revolving the line of action about the axis of the cam.

A conical cam might also be constructed to actuate a follower in a line perpendicular to its axis, and, by changing the position of the cone relatively to the follower (the cam sliding along its axis), a variation in the motion could be obtained. A plate cam might also be arranged to turn slightly relatively to its shaft, and the relative times of the motions would thus be changed, an arrangement made use of in some governing devices.

86. Two cams might be made, by means of a system of levers and rods, to act on the same point, the motions governed by each cam being in lines situated in one plane and making right angles with the other; the point so governed could be made to trace any plane curve within the limits of its movements.

If a plate cam is required to produce more than one double oscillation of a vibrating lever during one revolution, its edge would be formed into a corresponding number of waves: if a groove is used, the centre line of the groove would have the same series of waves.

87. DIAGRAMS FOR CAMS GIVING RAPID MOVEMENTS.—It is very often the case that a cam is required to give a definite motion in a short interval of time, the nature of the motion not being fixed. We will now discuss the form of the diagram for such a motion:

In the diagram, shown in connection with FIG. 1, PLATE 3, at *A*, the follower had two uniform motions, and if the cam be made to revolve quickly, quite a shock will occur at each point that the motion changes, as *a*, *b*, and *c*; to obviate this, the form of the diagram can be changed, provided it is allowable to change the nature of the motion.

Suppose we wish a cam to rapidly raise a body from *C* to *C'* (FIG. 5, PLATE 5), the nature of the motion to be such that the shock shall be as light as possible.

If we draw the straight line *CA*, we have the case of a uniform motion (as in FIG. 1, PLATE 3), the body being raised from *C* to *C'* in an interval proportional to *BC*, which is called the time line; here the motion changes suddenly at *C* and *A*, accompanied with a perceptible shock.

The line *ED* drawn above *CA* would be an improvement, the follower not needing so great an impulse as at the start, or at the end of the motion, both being much more gradual than before.

The body might have a *simple harmonic motion*, the diagram for which would be drawn as follows:—

Draw the semi-circle *B6A* on $AB = CC'$ as a diameter; divide the time line *BC* into a convenient number of equal parts (in this case 8), and then divide the semi-circle into the same number of equal parts; through the divisions of the semi-circle draw horizontal lines intersecting the vertical lines drawn through the corresponding points of division of the time line BC, as a' b' c', etc. A smooth curve drawn through these points gives the full curve $C\,a'\,b'\,c'\,d' \dots A$. Here the body or follower receives a velocity increasing from zero at the start to a maximum at the middle of its path, when it is again gradually diminished to zero at C', the end of its path.

This form of a diagram gives very good results, and is satisfactory in many of its practical applications. But it is a question whether a body can be lifted by its use through a given space in the shortest time without a shock.

88. A body dropped from the hand has no initial velocity at the start, but has a uniformly increasing velocity, under the action of gravity, until it reaches the ground. Similarly, if the body is thrown upward with the velocity it had on striking the ground, it will come to rest at a height equal to that from which it was dropped, and its upward motion is the reverse of the downward one, that is, a uniformly retarded motion.

It is reasonable to assume, in designing a cam, that the motion of the follower should follow the same law of gravity, and have a uniformly accelerated motion until the middle of its path is reached, then a uniformly retarded motion to the end of its path.

A body free to fall descends through spaces, during successive units of time, proportional to the odd numbers, 1, 3, 5, 7, 9, etc., and the total space passed over equals the sum of these spaces.

To develop a line of action according to this law upon the same time line BC, and with the same motion CC', Fig. 5, Plate 5, as before, proceed as follows (Fig. 6, Plate 5):

Divide the time line BC into any odd number of equal parts as eleven; then sum up the relative spaces to be passed over by the follower, increasing through six spaces, and diminishing by the same law through the remaining five, $1+3+5+7+9+11+9+7+5+3+1=61$, giving sixty-one in this case; next obtain the relative spaces passed over from the start, at the end of each interval, by adding the space passed over in the interval to all preceding spaces, the result of which is shown in the table at the right of the figure in column *h*, where the relative space passed over at the end of the second interval is four; of the third nine; and so on. Now divide the line *AB* into sixty-one parts, and mark the divisions corresponding to the numbers in column *h* as 1, 4, 9, 16, etc.; through these points draw horizontal lines, and note the points a'', b'', c'', etc., where they intersect the vertical lines drawn through the corresponding divisions of the time line; a smooth curve drawn through these points, gives the cam diagram.

As the motion above *O* is the counterpart of that below *O*, it will be noticed from the figure that the points 25 and 36, 16 and 45, etc., on *AB* are equidistant from the centre of motion *O*, which will enable us to save considerable work in dividing *AB*, one-half of which *BO*, would contain thirty and one-half of the divisions obtained before. To find the points 4, 9, 16, etc., draw a line, as *Bf*, passing through *B*, and space off upon it thirty and one-half equal divisions, as shown; through the centre *O* and the last division 30½ draw a line; and through the points 25, 16, 9, etc., in *Bf*, draw lines parallel to *O* 31½ thus obtaining the points 25, 16, 9, etc., on *BO*, from which 36, 45, 52, etc., can be found as shown.

Comparing this curve with that obtained from the harmonic motion, it will be seen that the latter gives too great acceleration during the first quarter of the motion, too little during the second quarter, too great a retardation in the third quarter and too little during the fourth quarter.

In designing cams to give rapid motions without shocks, the gravity curve should be used.

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PLATES VI AND VII.

Rolling Cylinders and Cones connected by Force-closure.

89. One of the most common and most useful problems of mechanism is to connect two axes so that they shall have a definite angular velocity ratio. We may have three cases: 1°, parallel axes; 2°, axes which intersect; 3°, axes which are neither parallel nor intersecting. In any case the velocity ratio may be 3 constant or it may vary. The first two cases, with a constant velocity ratio and directional relation, are the most common, and resemble each other in many ways; for that reason they will be considered first; the third case will come up later.

Pure rolling contact consists of such a relative motion of two lines of surfaces, that the consecutive points or elements of one come successively into contact with those of the other in their order.

90. Rolling cylinders.—Let A and B (Fig. 1, Plate 6) be the sections of two rolling cylinders, and O and O_1, the centres of the circles A and B, tangent at C, be the traces of their axes. The point C is called the *point of contact.* The common element through C is called the *line* or *element of contact.* The plane passing through the axes, whose trace is the common normal through the point C, is called the *plane of centres;* the common normal OO_1 is called the *line of centres.*

According to the definition of rolling contact, the linear velocity ratio of the two rolling surfaces must be unity, or slipping will result between them.

91. Angular velocity ratio.—In Fig. 1, Plate 6, let R and r be the radii of two rolling cylinders, V and v their angular velocities, and N and n their revolutions per minute. Suppose the two cylinders to roll in such a way that the point C travels to F in A, and to E in B; then, as no slipping can occur, and as the paths of the common point C on each circle, are the measures of the linear velocity in each case, and as the linear velocities of the surfaces of the rolling cylinders must be equal,

$$\text{arc } CF = \text{arc } CE, \text{ or } RV = rv.$$

$$\therefore \frac{V}{v} = \frac{r}{R} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

That is, when two cylinders or circles roll together, their uniform angular velocities are inversely as their radii.

Now if, as is more commonly the case, we take the number of revolutions per minute, N and n, as given, the linear velocities per minute would be $2\pi RN$ and $2\pi rn$, which must be equal.

$$\therefore 2\pi RN = 2\pi rn, \text{ or } RN = rn.$$

$$\therefore \frac{N}{n} = \frac{r}{R} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

That is, when two cylinders or circles roll together, the numbers of revolutions of each in a given time are inversely proportional to their radii.

Given the velocity ratio and the distance between the centres of a pair of rolling cylinders, to find the radii.

92. EXTERNAL CONTACT (FIG. 1, PLATE 6).—If D is the distance between the axes, and A makes N revolutions and B makes n revolutions per minute, we have from equation (2)

and from the figure,
$$\left.\begin{aligned} \frac{r}{R} &= \frac{N}{n} \\ R + r &= D \end{aligned}\right\}$$

Solving the two equations for R and r, we have

$$R = \frac{nD}{n+N}, \text{ and } r = \frac{ND}{n+N}$$

93. INTERNAL CONTACT (FIG. 2, PLATE 6).—Using the same notation as above, we have

and
$$\left.\begin{aligned} \frac{r}{R} &= \frac{N}{n} \\ R - r &= D \end{aligned}\right\}$$

$$R = \frac{nD}{n-N}, \text{ and } r = \frac{ND}{n-N}$$

Since $V = 2\pi N$ and $v = 2\pi n$, we have

$$R = \frac{vD}{v \pm V}, \text{ and } r = \frac{VD}{v \pm V}$$

the positive sign being taken for external, and the negative sign for internal contact.

94. GRAPHICAL SOLUTION.—*External contact.* Let OO_1 be the line of centres (FIG. 3, PLATE 6). Through O draw the line ON, and lay off upon it, on some convenient scale, the distances OM and MN equal to the number of units in n and N respectively. Draw NO_1 through the centre O_1, and then draw MC parallel to NO_1. C will be the point of contact, and R and r the radii.

Internal contact, FIG. 4, PLATE 6.—Draw the line ON as before, and lay off OM equal to the units in n; then lay off MN toward O from M, equal to the units in N; draw NO_1 and MC parallel to it; R and r will be the radii, and C the point of contact.

It will be noticed that two cylinders in external contact revolve in opposite directions, while those in internal contact revolve in the same direction.

95. RACK AND PINION.—If we suppose the circle B (FIG. 1, PLATE 6) to be indefinitely increased, its radius will eventually become infinite, and its circumference will become sensibly a straight line. This combination is often used in "gearing", and the velocity ratio is properly considered here. As the straight line or rack has straight translation only, its linear motion needs only to be considered. If the radius R, and the angular velocity, or number of revolutions N, of the rolling circle or pinion are given, the linear velocity of the rack, L, is found by the equation

$$L = 2\pi RN.$$

For, to satisfy the conditions of rolling contact, the linear velocities must be equal. Here the velocity ratio is constant, but the directional relation must be reversed when the end of the rack reaches the pinion.

96. ROLLING CONES.—Let A and B (FIG. 5, PLATE 6) be a pair of rolling cones; CE, the element of contact; OCO_1E, the plane of centres. Let OCS and O_1CT be planes passing through the point of contact C, and respectively perpendicular to OE and O_1E; the circles cut from the cones by these planes are called *base circles.* Similar circles could be cut by planes parallel to OCS and O_1CT through any other point on the element of contact, as c. Now in order to have no slipping, the linear velocity

ratio of all pairs of points on the common element of contact *CE* must be unity, that is, the consecutive elements of one cone must roll successively into contact with those of the other.

97. ANGULAR VELOCITY RATIO.—Let R and r be the radii of the base circles, N and n be their revolutions per minute, and V and v their angular velocities. Since the linear velocities of the two base circles in contact at C are the same, we have for rolling circles,

$$\frac{R}{r}=\frac{v}{V}=\frac{n}{N}$$

and for any other point, as c, we must also have

$$\frac{R_1}{r_1}=\frac{v}{V}=\frac{n}{N}$$

as R and R_1 must revolve with the same angular velocity about *OE*, they being in one piece. It can also easily be seen by similar triangles, that

$$\frac{R}{r}=\frac{R_1}{r_1}$$

Thus the angular velocity ratio of any two circles in contact on the common element is the same as that of the base circles, and that of the two axes.

In practice it is customary to use thin frusta of the rolling cones for rolling conical wheels, as shown in FIG. 5, PLATE 6; the dimensions of the frusta depend upon their distance from the intersection of the axes.

Given the angular velocity ratio, the position of one conical wheel, and the angle between the axes: to find the position of the other wheel, and the radii of both wheels.

98. ALGEBRAICAL SOLUTION.—Let *OE* aud O_1E (FIG. 6, PLATE 6) be the axes of a pair of revolving cones, θ the angle between the axes, R and r the radii, and α and β the semi-vertical angles.

$$R=EC\sin\alpha$$

$$r=EC\sin(\theta-\alpha),$$

$$\frac{\sin\alpha}{\sin(\theta-\alpha)}=\frac{R}{r}=\frac{n}{N}$$

$$\therefore\frac{n}{N}=\frac{\sin\alpha}{\sin\theta\cos\alpha-\cos\theta\sin\alpha}$$

$$=\frac{\dfrac{\sin\alpha}{\cos\alpha}}{\sin\theta-\cos\theta\,\dfrac{\sin\alpha}{\cos\alpha}}$$

$$=\frac{\tan\alpha}{\sin\theta-\cos\theta\tan\alpha}$$

$$\therefore\ \tan\alpha=\frac{n\sin\theta}{N+n\cos\theta}=\frac{\sin\theta}{\dfrac{N}{n}+\cos\theta}$$

The equation

$$\tan\beta=\frac{\sin\theta}{\dfrac{n}{N}+\cos\theta}$$

may be derived in the same manner.

Now, the angles α and β being known, and the distance OE of one base circle from the intersection of the axes being given, the distance O_1E of the other may be found thus:

$$EC=\frac{OE}{\cos\alpha}\text{, and }O_1E=EC\cos\beta=OE\frac{\cos\beta}{\cos\alpha}$$

The radii of the base circles may be found as follows:

$$R=OE\tan\alpha=\frac{OE\sin\theta}{\dfrac{N}{n}+\cos\theta}$$

$$r=O_1E\tan\beta=\frac{O_1E\sin\theta}{\dfrac{n}{N}+\cos\theta}$$

or, since

$$\frac{r}{R}=\frac{N}{n}\text{, whence }r=\frac{N}{n}R,$$

$$r=\frac{OE\sin\theta}{1+\dfrac{n}{N}\cos\theta}$$

which is a more convenient form to use.

The angle θ between the two axes is often 90°, in which case the preceding equations reduce to

$$\tan\alpha=\frac{n}{N};\ \tan\beta=\frac{N}{n};\ R=OE\frac{n}{N};\ r=OE.$$

CE may be found from the equation $CE=\sqrt{(\overline{CE}^2+R^2)}$

99. GRAPHICAL SOLUTION.—On any convenient scale lay off the distance ED (FIG. 6, PLATE 6)$=N$; draw DF parallel to EO_1; with the same scale lay off $DF=n$, and F will be a point on the common element EFC. Now draw OC perpendicular to OE, and CO_1 perpendicular to O_1E. As a proof of the preceding construction, we have for the triangle DEF,

$$\frac{ED}{DF}=\frac{\sin\beta}{\sin\alpha}=\frac{r}{R}=\frac{N}{n}$$

Another method is as follows: On some convenient scale draw a line FG parallel to, and at a perpendicular distance $Fn=n$ from OE; and a line DF parallel to, and at a perpendicular distance $FN=N$ from O_1E; the point F, where the two lines intersect, is a point on the common element; finish as before.

Here

$$\frac{\sin\beta}{\sin\alpha}=\frac{N}{n}=\frac{r}{R}$$

In the above formulæ V may be substituted for N, and v for n, as $\frac{N}{n}=\frac{V}{v}$, thus giving a similar series of formulæ in which the angular velocities appear.

100. FRICTION GEARING.—Rolling cylinders and cones are frequently used to transmit force, and constitute what is known as friction gearing. In such cases the axes are arranged so that they can be pressed together with considerable force, and, in order to prevent slipping, the surfaces of contact are made of slightly yielding materials, such as wood, leather, rubber or paper, which, by their yielding, transform the line of contact into a surface of contact, and also compensate for any slight irregularities in the rolling surfaces. Frequently only one surface is made yielding, the other being usually made of iron.

As there is often slipping in the above combination, the velocity ratio cannot be depended upon as absolute.

When rolling cylinders or cones are used to change sliding to rolling friction, that is, to reduce friction, their surfaces should be made as hard and smooth as possible.

101. GROOVED FRICTION GEARING.—Another form of friction gearing is shown in FIG. 7, PLATE 6. Here increased friction is

obtained between the rolling bodies by supplying their surfaces of contact with a series of interlocking angular grooves; the sharper the angle of the grooves, the greater the friction for a given pressure perpendicular to the axes, both wheels are usually made of cast iron. Here the action is no longer that of rolling bodies; but considerable sliding takes place, which varies with the shape and depth of the groove. This form of gearing is very generally used in hoisting machinery for mines, and also for driving rotary pumps; in both cases a slight slipping would be an advantage, as shocks are quite frequent in starting suddenly, and their effect is less disastrous when slipping can occur.

The velocity ratio is not absolute, but is substantially the same as that of two cylinders in rolling contact on a line *CE*, drawn midway between the tops of the projections on each wheel, they being supposed to be in working contact.

Levers.

102. We often have occasion to transfer some small motion from one line to another; we will consider three cases depending on the relative positions of the lines of motion:

1°, Parallel lines.

2°, Intersecting lines.

3°, Lines neither parallel nor intersecting.

In 1° we have the common lever *AB* (Fig. 8, Plate 6) where, when the lever has a small angular motion, the points *A* and *B* receive motions proportional to their distances from the centre *C*, and approximately in the parallel lines *Aa* and *Bb*. If we suppose the lever to move through the small angle *a*, we have, since the angular velocity is proportional to the angular space swept over, and the linear velocity equals the product of the angular velocity by the radius,

$$\frac{\text{Velocity of } A}{\text{Velocity of } B} = \frac{A\,C a}{B\,C a} = \frac{AC}{BC}$$

If *P* and *W* denote the forces applied at *A* and *B* respectively, we have, if they are at equilibrium,

$$P \times AC = W \times BC, \text{ or } \frac{P}{W} = \frac{BC}{AC}$$

or the forces are inversely proportional to the lengths of their lever-arms.

103. BELL-CRANK LEVER.—In 2° we have the bell-crank lever *ACB* (FIG. 9, PLATE 6), where *A* and *B* move approximately in the lines *AD* and *BD* intersecting at *D*. Here, as in 1°, the velocities are proportional to the lengths of the lever-arms, or are proportional to the sines of the angles *ADC* and *DCB*, made by the line *CD* with the directions of the motions.

Suppose the angle *ADB* to be given, and a bell-crank lever is required that will give a motion along *AD* equal to one-half that along *BD*.

Draw the line *DC* dividing the angle *ADB* into two parts whose sines are directly proportional to the velocities of *A* and *B*. This may be done by erecting perpendiculars on *AD* and *BD* in the ratio of the required motions along those lines, and drawing through their extremities the lines *ab* and *cd* parallel to *AD* and *BD* respectively; the intersection of these lines *E* determines the line *DE;* in this example *cd* is twice as far from *BD* as *ab* from *AD*. Choose any point *C* in *ED*, and drop the perpendiculars *CA* and *CB* on *AD* and *DB* respectively, then *ACB* is the required bell-crank lever.

Here, as in the previous case, if we suppose the lever to move through the small angle a, we have

$$\frac{\text{Velocity of } A}{\text{Velocity of } B} = \frac{AC \times a}{BC \times a} = \frac{AC}{BC} = \frac{\sin CDA}{\sin CDB},$$

It is evident that, for a small angular motion, the movements in *DA* and *DB* are very nearly rectilinear, and they will become more and more so, the farther we remove *C* from the point *D*.

Any slight motion that may occur perpendicular to the lines *DA* and *DB* may be provided for by the connectors used; that is, by making slots in the arms *AC* and *BC* for the pivots at *A* and *B*, and providing guides for the moving pieces, constraining them to move in the directions *AD* and *BD*. It is to be noticed, however, that for a given motion of the lines *DA* and *DB* these perpendicular movements, or deviations, are less when the lever-arms vibrate equal angles each side of their positions when perpendicular to the lines of motion, and they should always be arranged to so vibrate. By moving the point C nearer to D, at the same time keeping the lever-arms the same, this perpendicular deviation could be made to come equally on each side of the lines of motion, which is advisable, especially in cases where the deviation is allowed for by the spring of the connecting piece.

In 3° we have the lines of motion AD and BE (FIG. 10, PLATE 6) neither parallel nor intersecting. Draw PQ, the common perpendicular to the lines AD and BE; through Q draw QH parallel to DA, construct a bell-crank lever acb, for the movements considered as transferred to the lines BQ and HQ in one plane; draw ce parallel to PQ and equal to it, and further make ed parallel and equal to cb. The piece $aced$ will be the lever required.

What has been done is this: a bell-crank lever has been constructed in a plane, passing through the first line of motion and parallel to the second line; the transferred second line of motion in this plane being a projection of the second line of motion; or the lever acb has been arranged to transfer the motion from BE to QH; then the motion along QH has been transferred to the second line AD parallel to QH by moving the lever cb parallel to itself an amount equal to the perpendicular distance between the two lines of motion, and connecting e and c by means of a shaft, so that ed and ac turn together about ce. The rocker arm on a locomotive, which transfers the motion from the link inside of the engine frame, to the valve rod, outside of the engine frame, is an example of this form of lever.

Belts and Pulleys.

104. A flexible wrapping connector may be paired by force-closure with a pulley, and two such pairs may be combined to connect two axes, whether parallel, intersecting, or neither parallel nor intersecting. Flexible connectors may be divided into three general classes:—

1. Belts made of leather, rubber, or woven fabrics, are flat and thin, and require pulleys nearly cylindrical with smooth surfaces. Flat ropes may be classed as belts.

2. Cords made of catgut, leather, hemp, cotton, or wire are

nearly circular in section, and require either grooved pulleys or drums with ledges. Rope gearing, either by cotton or wire rope, may be placed under this head.

3. Chains are composed of links or bars, usually metallic, jointed together, and require wheels or drums either grooved, notched, or toothed, so as to fit the links of the chain. For convenience the word band may be used as a general term to denote all kinds of flexible connectors.

Bands for communicating continuous motion are endless.

Bands for communicating reciprocating motion are usually made fast at their ends to the pulleys or drums which they connect.

105. THE LINE OF CONNECTION of a pair of pulleys connected by a band is the central line or axis of that part of the band which transmits the motion.

THE PITCH SURFACE of a pulley over which a band passes is the surface to which the line of connection is always tangent; that is, an imaginary surface whose distance from all parts of the acting surface of the pulley is equal to the distance from the acting surface of the band to its neutral or centre line. The pitch surface of a pulley cannot be concave, as the belt will always stretch across a hollow in a straight line. Belts are usually made to transmit a nearly constant and continuous velocity ratio, and in this case the acting surfaces are cylindrical.

THE EFFECTIVE RADIUS of a pulley is the radius of its pitch surface, and is equal to the radius of the pulley plus one-half the thickness of the band.

THE PITCH LINE of a pulley is the line on its pitch surface in which the centre line of that part of the band which touches the pulley, lies. The line of connection is tangent to the pitch line.

106. VELOCITY RATIO.—If we assume the band to be inextensible, and that there is no slipping between it and the pulley, the linear velocity of the pitch surfaces of the connected pulleys must be equal. Let V be the velocity of the band, d_1 and d_2 the diameters of the two pulleys which it connects, and N_1, N_2, their revolutions per minute respectively, the thickness of the band being δ; then

$$V = \pi N_1(d_1+\delta) = \pi N_2(d_2+\delta).$$

✓

$$\therefore \frac{d_1+\delta}{d_2+\delta}=\frac{N_2}{N_1}$$

That is, the angular velocity ratio, as in rolling cylinders, is inversely proportional to the effective diameters, and is constant for circular pulleys. As the thickness of thin flat bands or belts is generally small compared with the diameters of the pulleys connected, it may be neglected, and the acting surfaces of the pulleys may then be considered to have the same linear velocity. The approximate angular velocity ratio will then be given by the equation:

✓

$$\frac{d_1}{d_2}=\frac{N_2}{N_1} \quad . \quad . \quad . \quad . \quad (3)$$

Belts and cords are not suited to transmit a precise velocity ratio, because they are liable to stretch or to slip on the pulleys. This freedom to stretch and slip is an advantage in powerful and quick running machinery, as it prevents shocks which are liable to occur when a machine is thrown suddenly into gear, or when there is a sudden fluctuation in the power transmitted.

107. Crowning of pulleys.—If we suppose a belt to run upon a revolving conical pulley, it will tend to lie flat upon the conical surface, and, on account of its lateral stiffness, will assume the position shown in Fig. 1, Plate 7. If the belt travels in the direction of the arrow, the point *A* will, on account of the pull on the belt, tend to adhere to the cone and will be carried to *B*, a point nearer to the base of the cone than that previously occupied by the edge of the belt: the belt would then occupy the position shown by the dotted lines. Now if a pulley is made up of two equal cones placed base to base, the belt will tend to climb both, and would thus run with its centre line on the ridge formed by the union of the two cones. In practice pulley rims are made crowning, except in cases where the belt must occupy different parts on the same pulley. In Fig. 2, Plate 7, two common forms of rim sections are shown.

When pulleys are improperly located, the belt will run toward the tight side of the pulley; this is due to the lateral stiffness of the belt, and could be explained in the same way as the climbing on a conical pulley.

108. TIGHT AND LOOSE PULLEYS are used for throwing machinery into and out of gear. They consist of two pulleys placed side by side upon the driven shaft *CD* (FIG. 3, PLATE 7); *A*, the tight pulley, is keyed to the shaft; while *B*, the loose pulley, turns loose upon the shaft, and is kept in place by the hub of the tight pulley and a collar *aa*. The driving shaft carries a pulley *G*, whose width is the same as that of *A* and *B* put together, or twice that of *A*. The belt, when in motion, can be moved by means of the shipper *S* (FIG. 4, PLATE 7) that guides its advancing slide, either on to the tight or the loose pulley. The tight pulley is sometimes made larger than the loose pulley, so the belt may be slack when it is on the loose pulley. In such a case the loose pulley has a ledge upon the edge next the tight pulley, of a diameter equal to that of the tight pulley; this ledge aids in the transfer of the belt from one pulley to the other.

The acting surface of a pulley is called the face; a pulley with a six-inch face is one having a face six inches wide. The faces of pulleys are always made a little wider than the belts which they carry. The pulley *G* (FIG. 3) has a flat face, because the belt must occupy different positions upon it; while *A* and *B* may have crowning faces, which will allow the shifting of the belt, and will retain it in position when shifted upon them. In practice *A* and *B* often have flat faces.

109. SPEED CONES are contrivances for varying and adjusting the velocity ratio of a pair of rotating parallel axes by means of a shifting belt: They may be continuous cones or conoids as in FIG. 5, PLATE 7, where the velocity ratio can be varied gradually, while they are in motion, by shifting the belt, or they may be sets of stepped pulleys as in FIG. 6, PLATE 7, where by shifting the belt, only a certain number of distinct speeds can be imparted at will to the follower, depending on the number of steps. In either case the length of the belt must not vary, in order that it may be equally tight in all positions. For length of belt and discussion of speed cones see Art. 56 and 57, Schwamb's *Elements of Mechanism.*

Speed cones are used in nearly all cutting tools of precision, where different speeds are required for the different classes of work done.

Lathes, bolt cutters and drills all have speed cones, or stepped pulleys more generally, to vary the speed, according to the character of the work, or perhaps according to the degree of hardness of the material being worked.

110. Pulleys for round ropes or cords must be provided with V-grooves to keep the ropes in place: any flat bands not having sufficient lateral stiffness would also require ledges to keep them on the pulley.

Small cords are used to transmit small amounts of power, as for driving the spindles in spinning machinery.

Hemp and cotton ropes are now very generally used in England to transmit quite large amount of power; when so used they are generally run in sets, each pair of pulleys carrying several ropes. The grooves are made V-shape, and the ropes are drawn into them, thereby increasing the hold upon the pulleys. Rope pulleys are usually made of cast iron, and the grooves are turned in a lathe. FIG. 8, *A*, PLATE 7, shows a section of the rim of a pulley for carrying a number of ropes.

Large amounts of power are now successfully transmitted by rapidly moving wire ropes carried by large grooved wheels. Wire ropes will not support without injury the lateral crushing due to the V-shaped grooves: hence it is necessary to construct the pulleys with grooves so wide that the rope rests on the rounded bottom of the groove, as shown in FIG. 8, *B*, PLATE 7, which shows a section of the rim of a wire rope pulley.

111. EFFECTIVE PULL.—By the effective pull on a belt, we mean the pull that is doing the work. If in FIG. 3, PLATE 7, we assume A the driver, and G the follower, T_1 the tension in the tight, and T_2 the tension in the loose belt, the effective pull will be $(T_1 - T_2) = P$. Suppose A to be 30″ in diameter, to make 200 revolutions per minute, and to carry a belt transmitting 4 horse-power or 4 × 33,000 foot-pounds per minute; what is the effective pull of the belt?

Here the work done by the belt, that is, the effective pull on the belt, in pounds, multiplied by its speed in feet per minute, must equal the horse-power transmitted multiplied by 33,000; or

$$\frac{30 \times 200 \times 3.1416}{12} \times P = 4 \times 33000.$$

$$P = 84.04 + \text{lbs.}$$

The tight side of a belt is the one doing the work, and is the one moving toward the engine.

From the above example it can readily be seen that the quicker a belt travels, the smaller the pull for a given horse-power transmitted.

The plane of a pulley is a central plane through the pulley perpendicular to its axis, or is the plane of the pitch line of the pulley. The point of delivery of a pulley is the point in the pitch line at which the belt leaves the pulley.

The general principle governing the arrangement of belt pulleys upon non-parallel axes may be stated as follows: The belt must always be delivered in the plane of the pulley toward which it is running. That is, a belt leaving a pulley may be drawn out of the plane of the pulley; but when approaching a pulley, its centre line must lie in the plane of the pulley.

112. GUIDE PULLEYS are used to change the directions of belts and are placed according to the above rule. It is possible by the use of two guide pulleys, to connect any pair of pulleys by an endless belt, and the guide pulleys may be so placed that the belt will travel in either direction, which is sometimes a great advantage.

BINDER PULLEYS.—Guide pulleys are often used, as in FIGS. 7 and 7*a*, PLATE 7, to increase the arc of contact of the belt, and also, as there shown, to tighten the belt; when so used, they are called binder or tightening pulleys, and are always applied to the loose belt.

Pulleys for belts could be arranged in many other ways, but the same principles govern the arrangement in each case. When convenient, it is best to arrange the belt to travel in either direction, as engines are often moved backward a part of a turn, thus rendering any belt not admitting of motion in either direction liable to be thrown off.

Cords and ropes, especially wire ropes, are in general only used to connect parallel axes, but when otherwise used, the same rules as were given for flat bands govern the arrangement. As ropes have no lateral stiffness, they are not used to connect vertical axes without supplying guide pulleys to insure the proper guiding of the ropes into the grooves.

113. DRUM OR BARREL.—When a cord does not merely pass over a pulley, but is made fast to it at one end, and wound upon it, the pulley usually becomes what is called a drum, or barrel. A drum for a round rope is cylindrical, and the rope is wound upon it in helical coils. Each layer of coils increases the effective radius of the drum by an amount equal to the diameter of the rope. A drum for a flat rope is of a breadth simply equal to the breadth of the rope, which is wound upon it in single coils, each of which increases the effective radius by an amount equal to the thickness of the rope.

Small cords are often used to connect non-parallel axes, and very often the directional relation of these axes must vary. The most common example is found in spinning frames and mules, where the spindles are driven by cords from a long, cylindrical drum, whose axis is at right angles to the axes of the spindles. In such cases, the common perpendicular to the two axes must contain the planes of the connected pulleys; both pulleys may be grooved, or one may be cylindrical, as in the example given above. FIG. 9, PLATE 7, shows two grooved pulleys, whose axes are at right angles to each other, connected by a cord which can run in either direction, provided the groove is deep enough. To determine whether a groove has sufficient depth in any case, the following construction (FIG. 10, PLATE 7) may be used. Let *ab* and *a'b'* be the projections of the approaching side of the cord; pass a plane through *ab* parallel to the axis of the pulley; it will cut the hyperbola *Cbd* from the cone *feg*, which forms one side of the groove. The cord will lie upon the pulley from *b* to *i*, where it will leave the hyperbola on a tangent. If the tangent at *i* falls well within the edge of the pulley *C*, the groove has depth enough. It will usually be sufficient to draw a straight line, as *ab* (FIG. 9), and see that it falls well inside of the point corresponding to *C* in FIG. 10.

114. In cases where a considerable amount of power has to be transmitted between two shafts running at a slow speed, metal chains, called gearing chains or pitch chains, are used with toothed pulleys. When a chain is to drive or to be driven by a

pulley, or sheave, as it is sometimes called, the acting surface of the pulley must be adapted to the figure of the chain, so as to insure sufficient hold between them. Guide pulleys for chains are made circular, and grooved to suit the form of the chain. Chain drums or barrels have one end of the chain made fast to them, and the chain is guided by a suitably formed helical groove. Such drums are used in cranes.

Gearing chains are usually made of flat links, riveted or pinned together at their ends, so as to allow a free turning of the links at the joints. The most simple form consists of double and single links arranged alternately, the single link being made thick enough to have the same strength as the double links.

A geared chain formed of square open links made of malleable iron is now very extensively used in agricultural and other machinery requiring light geared chains. Fig. 11, *A*, Plate 7, shows a side elevation of the chain and pulley, and at *B* the form of the link and the method of fastening are shown. The sides and one end of each rectangular link are round in section, and the remaining end carries a hook-shaped projection which is as wide as the rectangular opening, and hooks over the rounded end of the adjacent link; the rectangular spaces between the hooks and opposite ends of the links receive the teeth of the wheel, which act against the rounded inner parts of the hooks as on round pins.

Very large chain wheels may have cylindrical faces provided with suitably formed projections or hollows.

The chief objections to chain gearing, are the liability to stretching of the links forming the chain, and the wearing of the pins, or links, both of which tend to alter the pitch of the links, and thus cause bad fitting between the chain and wheel. The necessity for slow motion is also sometimes a disadvantage.

Wheels and chains of the general type illustrated in Fig. 11, Plate 7, are sometimes called sprocket wheels and sprocket chains.

PLATES X AND XI.

Pulleys.

115. Plate 10, Fig. 1 illustrates a general case of guide pulleys. Let *AA′* and *BB′* be the vertical and horizontal projection of a pair of pulleys to be connected by a flat belt. The axes of these pulleys are not parallel nor in the same horizontal plane. The planes of the pulleys intersect on the line *SS′*. Choose *a* and *c*, convenient points on the line *SS′*; from *a*, draw a line *ab*, and from *c*, draw a line *cd*, both in the plane of the pulley *A* and tangent to the pitch line of *A*.

Similarly draw *ae* and *cf* in the plane of the pulley *B* and tangent to its pitch line. The two lines *ab* and *ae* determine the plane of the guide pulley *C*, it being so placed that its pitch line is tangent to both *ab* and *ae*.

In this position it can take a belt delivered from either *A* or *B* and delivered at *e* or *b*, as both *e* and *b* are in the plane of the guide pulley *C* by construction. It can also deliver a belt to *A* or *B*, as it has the points of delivery in the planes of *A* and *B*. The position of the guide pulley *D* is similarly determined by the tangents *dc* and *cf*.

D and *C* being thus located, *A* and *B* can be connected by a belt, running over *C* and *D* as shown, as the same side of the belt comes in contact with all the pulleys, it is immaterial which way it runs. As double bending, or bending back and forth tends to injure a belt, it is desirable, whenever possible, to have the same side of the belt come in contact with each of the pulleys.

116. When a belt always travels in the same direction, it is only necessary to provide for its running properly in that direction, and in such a case the belt must always be delivered in the plane of the pulley toward which it is running.

Fig. 2, Plate 10, shows a quarter-turn belt connecting two pulleys *A* and *B* whose axes are in parallel planes and at right angles with each other. *SS′* (Figs. 2 and 3) is the trace of the intersecting planes of *A* and *B*, and if we suppose the pulley *A* to revolve in the direction indicated by the arrow, it delivers the belt at *a* in the plane of *B*, while *B* delivers the belt at *c*′ (FIG. 4) in the plane of the pulley *A*; thus rotation indicated by the arrows is allowable. If, however, we attempt to turn *A* in the opposite direction, the belt will immediately leave the pulleys, for *d*, the point of delivery of *B*, is not in the plane of *A*, nor *e* in the plane of *B*. See *d*′, FIG. 4. If the pulley *B*, FIG. 5, with its plane is swung about *SS*′ as an axis into any position such as *B*″, FIG. 5, the belt will still run in the direction indicated by the arrow, as the same conditions exist as before. In fact, if we draw on a piece of paper the pitch lines of two pulleys as *A* and *C* (FIG. 3), and draw a line *SS*′ tangent to them the paper may then be folded on *SS*′, and it can easily be seen that the point of delivery *a* of the pulley *A* is in the plane of *C*, and that the point of delivery of *C* is in the plane of *A*, no matter how the plane of *C* is turned about *SS*′.

117. It is not well to use quarter-turn belts where there is any liability of the motion being reversed, as when this happens they will immediately leave the pulleys. The angle at which the belt is drawn off from the pulleys should not be great, as when this is the case the belt is much strained, and does not hug the pulley well; power is also lost in side slipping. To make this angle as small as possible, the pulleys should be placed a sufficient distance apart, and they should be as small in diameter and carry as narrow a belt as practicable.

118. FIGS. 6 AND 7, PLATE 10, show a quarter-turn belt arranged with a double pulley so as to run in either direction; *A* is the driving pulley and *B* is the driven pulley, which is arranged for two positions of the belt.

119. Two shafts at right angles to each other, are often connected by means of a belt and one guide pulley, as shown in FIG. 7, PLATE 7. Here *ab* is the trace of the two intersecting planes of the pulleys *A* and *B* to be connected, and the plane of the guide pulley *C* is found, by taking a point in *ab*, as *c*, and from it drawing lines *ce* and *cd* tangent to the pitch lines of *A* and *B*, respectively. These two lines *ce* and *cd* determine the plane of the guide pulley *C*. Whenever a guide pulley is used, as herein described, it should not be placed so as to bear on the working

part of the belt which passes from *a* to *b* (FIG. 7, PLATE 7), but under the other part from *e* to *d*, as the extra friction due to the working pull is saved on the guide pulley bearings.

120. Two axes that are in the same horizontal plane and intersect, or are in two horizontal planes one above the other, may be connected by means of a belt and two guide pulleys on a common vertical shaft, on which they turn loosely, being held in place by collars properly placed. The name of *mule pulleys* is often applied to guide pulleys arranged on a vertical shaft. FIG. 8, PLATE 10, shows in elevation and plan, two shafts at right angles to each other and in the same horizontal plane connected by means of a belt and mule pulleys. In order that a belt may run properly on pulleys having other than horizontal axes, they are made more crowning; and when the axes carrying the guide pulleys are vertical, the pulleys have flanges; or better, circular discs of greater diameter than the pulleys, are carried by the shaft on which the pulleys revolve; these discs are so located to serve as flanges.

121. FIG. 9, PLATE 10, shows these two forms of flanges, on the pulleys carried by the vertical shaft *SS'*; *AB* is of the first type, cast with the flange, part of the pulley; *CD* is of the second type; *FF* is the disc, carried by the shaft *SS'*, and held in place by a collar *K*, this disc is separate from *CD*, but nevertheless serves the purpose of a flange for *CD* and keeps the belt in place.

The connected shafts *TT'* and *SS'* may make any angle with each other, and still be connected by a belt and mule pulleys, provided they are in, or nearly in the same horizontal plane.

Shafts and Pulleys.

122. In the location of shafts that are to be connected with each other by belts, care should be taken to secure a proper distance one from the other. It is not easy to give a definite rule as to what this distance should be. Some authorities give this rule: Let the distance between the shafts be ten times the diameter of the smaller pulley. But while this is correct for some cases, there are many other cases in which it is not correct. Circumstances generally have much to do with the arrangement; and the engineer or machinist must use his judgment,

making all things conform, as far as may be, to general principles. This distance should be such as to allow a gentle sag of the belt when in motion.

123. A general rule may be stated thus: Where narrow belts are to be run over small pulleys, fifteen feet is a good average. For larger belts working on larger pulleys, a distance of twenty to twenty-five feet does well. Shafts on which very large pulleys are to be placed for main or driving belts should be twenty-five to thirty feet apart. In shafting located as above stated the belts work in a very satisfactory manner, the slack side while in motion, having a sag of 1½ to 2 inches on the short distance above mentioned; while the larger belts show a similar sag of 2½ to 4 inches, the main belts working well with a sag of 4 to 5 inches.

124. If too great a distance is attempted, the weight of the belt will produce a very heavy sag, drawing so hard on the shafts as to produce great friction in the bearings, while at the same time the belt will have an unsteady, flapping motion which will destroy both the belt and the machinery.

125. The connected shafts should never, if possible, be placed one directly over the other, as in such case the belt must be kept very tight to do the work.

It is desirable that the angle of the belt with the floor should not exceed 45°. It is also desirable to locate the shafting and machinery so that belts shall run off from the shaft in opposite directions, as this arrangement will relieve the bearings from the friction that would result where the belts all pull one way on the shaft.

If possible, the machinery should be so planned that the direction of the belt motion shall be from the top of the driving to the top of the driven pulley.

126. All pulleys should be carefully centred and balanced on the shafting. Driving pulleys, on which are to be run shifting belts, should have a perfectly flat surface. All other pulleys should have a convexity in the proportion of about three-sixteenths of an inch to one foot in width. The diameter of the pulley should be as large as can be admitted, provided they will not produce a speed of more than 3300 feet of belt motion per

minute. When this speed has been obtained, the possible size of the pulley may be reduced in proportion to the speed greater than 3000 feet per minute, as this speed is considered the limit of economy to the belt. The pulley should be a little wider than the belt required for the work. It is also well to consider the possibility of adding, at some future time, more machinery than at first contemplated, and to make all needed provision for such possible increase, by making the main and counter-belts extra large. Such a course often proves a large saving of expense, or, what amounts to the same thing in the end, guarantees the machinery and belts against over-work. Every pulley not placed in a damp room should be covered with a good leather lagging, put on by an experienced workman. A pulley so covered is capable of much greater and better results, as the belt is not so likely to slip. Repeated experiments prove that the advantage of leather-covered over smooth iron pulleys is fully 33⅓ per cent.

127. The whole arrangement of shafting and pulleys should be under the direction of a mechanical engineer, or a machinist thoroughly competent for such work. Destruction of machinery and belts, together with unsatisfactory results in the business, is a common experience, which may in most cases be traced to want of knowledge and care in the arrangement of the machinery, and in the width and style of the belts bought and in the manner of their use, while manufacturers of belts are often blamed for bad results which are caused by the faulty management of the mill-owner himself.

128. Having properly arranged the machinery for the reception of the belts, the next thing to be determined is the length and width of the belts.

When it is not convenient to measure with the tape line the length required, the following rule will be found of service: Add the diameters of the two pulleys together, divide the result by two, and multiply the quotient by 3¼. Add the product to twice the distance between the centres of the shafts, and you have the length required, substantially.

129. The width of the belt needed depends on three conditions: 1st, the tension of the belt; 2nd, the size of the smaller

$$I = \frac{H.P. \times 33000}{T \times S}$$ ✓

S = [illegible] $= \frac{1}{2} \times \pi R$

pulley and the proportion of the surface touched by the belt; 3rd, the speed of the belt.

The average strain under which leather will break has been found by many experiments with various good tannages to be 3200 pounds per square inch of cross section. A very nice quality of leather will sustain a much greater strain. In use on pulleys, belts should not be subjected to a greater strain than one-eleventh their tensile strength, or about 290 pounds to the square inch of cross section. This will be 55 pounds average strain for every inch in width of single belt three-sixteenths inch thick. The strain allowed for all widths of belting, single, light double and heavy double, is in direct proportion to the thickness of the belt. This is the safe limit; for, if a greater strain is attempted, the belt is likely to be overworked, in which case the result will be an undue amount of stretching, tearing out the lace or hook holes, and damage to the joints. When the belt is in motion the strain on the working side will be greater than on the slack side; and the average strain will be one-half the aggregate of both sides.

130. The working adhesion of a belt to the pulley will be in proportion both to the number of square inches of belt contact with the smaller pulley, and also to the arc of the circumference of the pulley touched by the belt. This adhesion forms the basis of all right calculation in ascertaining the width of belt necessary to transmit a given horse-power. A single belt, three-sixteenths inch thick, subjected to the strain we have given as a safe rule, 55 pounds per inch in width, when touching one-half of the circumference of a turned iron pulley, will adhere one-half pound per square inch of the surface contact: while if it be a leather-covered pulley, the belt will adhere two thirds of a pound per square inch of contact. If the belt touched but one-quarter of the circumference of the pulley, the adhesion is only one-quarter pound to the square inch of contact with the iron pulley, and one-third pound on the leather-covered pulley.

131. The adhesion which one inch in width of the belt has on the pulley is the number of pounds which each inch in width of belt is capable of raising or transmitting. Multiplying this by the velocity of the belt in feet per minute will give the total num-

ber of pounds each inch in width will raise or transmit one foot per minute. Now multiply 33,000, the number of pounds raised by one horse-power one foot per minute, by the horse-power to be transmitted, and divide this product by the number of pounds which one inch in width will raise or transmit one foot per minute, and the quotient will be the width of the belt required.

These facts, reduced to working formulas, give the following:

Proportion of pulley touched.	Turned iron pulley.	Leather covered pulley.
One-quarter of circumference.	$\text{Width} = \frac{\text{H.P.}\times 33000}{\text{velocity in feet} \times \frac{1}{4} \text{ contact in inches}}$	$\text{Width} = \frac{\text{H.P.}\times 33000}{\text{velocity in feet} \times \frac{1}{3} \text{ contact in inches}}$
One-third of circumference.	$\text{Width} = \frac{\text{H.P.}\times 33000}{\text{velocity in feet,} \times \frac{1}{3} \text{ contact in inches}}$	$\text{Width} = \frac{\text{H.P.}\times 33000}{\text{velocity in feet} \times \frac{1}{2} \text{ contact in inches}}$
One-half of circumference.	$\text{Width} = \frac{\text{H.P.}\times 33000}{\text{velocity in feet} \times \frac{1}{2} \text{ contact in inches}}$	$\text{Width} = \frac{\text{H.P.}\times 33000}{\text{velocity in feet} \times \frac{2}{3} \text{ contact in inches}}$
Two-thirds of circumference.	$\text{Width} = \frac{\text{H.P.}\times 33000}{\text{velocity in feet} \times \frac{2}{3} \text{ contact in inches}}$	$\text{Width} = \frac{\text{H.P.}\times 33000}{\text{velocity in feet} \times \text{full contact in inches}}$

From this table it will be easy to calculate the width of belt required in all cases where the horse-power is known.

In all this calculation it must be borne in mind that the strain allowed on the belt is 290 pounds per square inch of cross section, so that the thickness as well as the width, must be considered; consequently, a double belt must be used where it is necessary to transmit greater power with a narrow belt. Again: to increase the driving power of a belt, the pulleys may be enlarged in circumference, thus increasing the speed of the belt. This can often be done to advantage, provided that the speed be not carried above the safe limit.

132. Having obtained the lengths and widths required, care should be taken, in the purchase of the belts, to get a proper article. Always avoid belting made from leather tanned with sulphuric acid, alum and other chemicals, by which the hide is rapidly cured and made thick but of coarse fibre. The leather should be pliable, of fine, close fibre, solid in its appearance, and of smooth polished surface. The character of the workmanship

should also be considered. The belting should be of substantially uniform thickness, and the laps thoroughly made and fastened.

Putting on Belts.

133. With a tri-square cut the ends of the belt perfectly true. Punch the holes exactly opposite each other in the two ends. The grain side of the belt should be run next to the pulley, and the belt should run off, not on, to the laps. Many good methods for fastening the ends of the belt together are in use; but undoubtedly the lacing is the best, as it is flexible like the belt, and runs noiselessly over the pulley.

It is of the last importance that this article be first-class. Nothing can be more expensive to a user of belts than poor lacing.

FIGURES 43 AND 44, PLATE 11, show correct methods of lacing belts. FIG. 44 being the side running next to the pulley where the strands are run straight in the belt, the diagonal strands being on the opposite side as shown in FIG. 43.

FIG. 45 illustrates the method of lacing with one row of holes, the dotted lines showing the crossing of the lacing on the outer side of the belts. It is much better to make two rows of holes, except in very narrow belts.

134. In punching a belt for lacing it is desirable to use an oval punch, the longer diameter of the punch being parallel with the length of the belt, so as to cut off as little leather as possible. There should be in each end of the belt two rows of holes, placed zig-zag. In a three-inch belt there should be four holes in each end, two in each row. A six-inch belt should have seven holes, four of them in the row nearest the end. A ten-inch belt should have nine holes, in the same form. The edge of no hole should come nearer the side of the belt than three-fourths of an inch, and not nearer to the end than seven-eighths of an inch. The second row should be at least 1¼ inches from the end. On wide belts these distances should be even a little greater.

135. Begin to lace in the centre of the belt, and take much care to keep the ends exactly in line, and to lace both sides with equal tension. The lacing should not be crossed on the side of the

belt that runs next to the pulley. One of the worst enemies the beltmaker has to meet, is the improper lacing or other fastening of belts. Put your inexperienced and unskilled man anywhere else, but never set him to putting on belts.

Care of Belts.

136. If the belt should need oiling or greasing, avoid everything of a pasty nature. The belt should be made pliable, not covered with a sticky substance.

In "taking up" belts, observe the same rules as in putting on new ones. Never add to the work of a belt so much as to overload it.

Ox-leather belts stand at the head of those of all other materials for the satisfaction of all demands on belts. No other belts will stand the shipper; cotton belts are weakened when wet; rubber belts are rotted when oiled, but leather will stand wet and dryness, cold and heat, and last a long time even when oil saturated.

If belts are to be endless, they should be put on and drawn together by "belt clamps" made for the purpose. If the belt is made endless at the belt factory, it should never be run on to the pulleys, lest the irregular strain spring the belt. Lift out one shaft, place the belt on the pulleys, and force the shaft back into place.

Do not allow oil to drip upon the belts. It destroys the life of the leather.

Leather belting cannot safely stand above 110° of heat.

Whenever there is dissatisfaction in the working of a belt, the matter should be taken promptly and vigorously in hand, as generally when the real facts are ascertained, it will be found that the trouble is not with the belt itself, but is on account of overwork, or some irregularity in the machinery.

Guide Pulleys.

137. The following information concerning various modes of Belt Driving is given, believing that those who have occasion to transmit power under peculiar and difficult conditions, will find these illustrations helpful. The numbers are for PLATE 11, *Book of Plates*.

FIG. 25. This shows a belt in actual use in J. & J. Hunter's Print Works, Hestonville, Philadelphia. The belt is 79 feet long and 24 inches wide, transmitting 60 to 70 horse-power. The main engine wheel makes 56 Rpm. The folds of this belt, as shown, are within four feet of each other. The working of this belt illustrates the great advantage of having the pulling side upon the under fold of the belt, and running the belt slack upon the upper fold. The belt in question is a very heavy one, it being 5-ply rubber, and this great weight enables the belt to hug the small pulley without slipping, notwithstanding it is so slack upon the upper fold. This additional slackness also enables the belt by reason of its sag to come in contact with a very large part of the pulley's circumference. Here the tension of the belt is produced by its weight rather than by direct strain, and the increased arc of circumference secured by reason of this sag gives it a much larger capacity for transmitting power than a belt would have if it were necessary to draw it straight so that the upper fold should be substantially a straight line between the pulleys.

FIG. 26 shows a flat belt transmitting power between two parallel shafts over guide pulleys, the shafts not being in the same plane. These shafts may be located in any direction from each other, the arrangement of guide pulleys being as illustrated here.

FIG. 27 illustrates a mode of transmitting power by a flat belt from one shaft to another in a case where the pulleys are too near together for direct driving, the belt being directed by guide pulleys. These shafts may be arranged in any direction from each other, and may, or may not, be parallel to each other. Here the smaller pulleys are the guide pulleys.

FIG. 28. In this illustration we have an example of transmission of power between two buildings which are not parallel to each other. By properly adjusting the guide pulleys, the belt may be made to drive shafts parallel to each building.

FIG. 29. This illustration shows a method by which two shafts can be driven by a belt from one drive pulley, making a twist upon two portions of the belt that come together. The drive is then the same as if it were an ordinary cross belt.

FIG. 30. This illustration shows two belts running upon the

same pulleys, one belt over the other, a method adopted by some in places where the width of the belt is not sufficient to transmit the necessary amount of power. Experience varies with reference to the utility of this plan. If it becomes necessary to use two belts in this way, the best results will be obtained by allowing the inner belt to run quite slack, and the outer belt to be quite taut. The belts will then run as indicated in the illustration. The drawing side of both belts will be taut and run together, while upon the slack side the inner belt will sag, as shown. In the case of the belts in FIG. 30, the driving pulley is 5 feet in diameter, making 75 Rpm., and the driven pulley 3 feet in diameter, and the inner belt at *a* has a sag of from 12 to 18 inches.

FIG. 31. Here two belts are run from the same pulley *A*, driving two shafts, *B* and *C*, the three shafts being so situated that the middle shaft is in line with the other two. This is with some a favorite method of transmitting power where the conditions are favorable for such an arrangement, and by its means one driving pulley will do the work otherwise done by two.

FIG. 32. The pulley *A*, from which the belt deflects, should have a wider face than *B*, in the proportion of 10 to 6, and should be more rounding on the face than is usual, and the pulleys should be as small as may be to do the work, and should be of nearly equal size.

About 25 per cent. of belt contact is lost when the belt makes a quarter turn, even when the pulleys are of the same size. It is noticed in the performance of a leather belt that the first 90° of lap on the pulley fit closely as in the ordinary straight belt arrangement; but in the second 90° about half the width of the belt is forced from contact with the pulley by the strain in the substance of the belt, due chiefly to its imperfect elasticity, and primarily to the oblique deflection of the fold which is leaving the pulley.

With a belt perfectly elastic the same amount of contact, if not more, can be obtained, as with the open belt, since the belt would adhere to the face of the pulley up to the line of departure the same in one case as in the other.

When two shafts are at or nearly at right angles with each other, and not in the same plane (FIG. 32), and it is desired to drive one from the other by two pulleys only and a connecting belt, experience has proved that certain conditions are necessary. In the first place, the distance between the near faces of the pulleys must not be less than four times the width of the belt. The pulleys *A* and *B* should be so placed that the belt will lead from the face of one to the centre of the face of the other, that is, so that a plane passing through the centre of the face of one pulley will be tangent to that part of the face of the other from which the belt is running.

The diagram gives the position and proper proportions referred to.

Holes for Quarter Turn Belts.

138. Draw on a level floor, with chalk line and tram, two full sized views of the pulleys and position of the floor through which belts are to pass, or lay them down on paper to a convenient scale, observing that, that fold of the belt which leaves the face of one pulley must approach the centre of the face of the other in a line at right angles to the axis of the latter. Completing the figure as shown in Fig. 33, the points of interesection *a b c* and *d* will indicate the places in the floor, *E F*, where the centres of both folds of the belt will pass when drawn tightly and at rest. The obliquity of the opening can best be obtained by trial of tape line or narrow belt applied to the pulley faces in position, passing through small trial holes in the floor. Allowance in the hole should be made for the sag or slack of belt. This is the usual safe and practicable shop method.

139. Fig. 34. *A* is a driving pulley on a horizontal shaft, and *B* a driving pulley on an upright shaft. Notice here the width of pulley *B*. As in case of Fig. 32, it must not be forgotten that pulleys used for quarter turn belts on perpendicular shafts must be wider than the belt requires. This belt runs over a tightener *C*, which is placed upon an angle right to receive a flat belt from pulley *B* and deliver it to pulley *A*. It not only acts as tightener, but also as guide pulley. The pulley *C* should run upon a short shaft set in grooved bearings, so that its position may be changed to take up the slackness of the belt as the belt stretches. Pulleys *A* and *B* should be round faced, and the pulley *B* should be very crowning.

140. Fig. 35 respresents a condition which is unusual, and still one may occur, because such a condition has already existed. This illustration was taken from a belt in operation in the Atlantic Works of Philadelphia, and is used to drive a 54-inch circular saw, the periphery of which was driven at the rate of 8,400 Fpm, the mandrel being at right angles to the driving shaft.

On the mandrel is a 12-inch pulley, and on the driving shaft, which runs horizontally 8 feet above, is a wooden drum 24 inches diameter, 8½ feet long, upon which the belt—a ten-inch heavy single leather, travelling 1800 Fpm traverses back and forth, following the reciprocating movement of the saw mandrel. The forward movement of the saw when cutting is at the rate of 60 Fpm, and the return movement 120 Fpm.

From some cause (centrifugal force, perhaps, or because the belt was new, and therefore not as pliable as it would otherwise have been) the centre of the pulley had to be set 8 inches out of the path of the vertical line from the periphery of the drum. (See cut.)

141. By the arrangement shown in Fig. 36, shafts may be run which are placed at right angles to each other. In this instance *E* is the driving shaft. On it is placed a tight pulley *A* and a loose pulley *B*. *F* is the driven shaft, and on it is placed a tight pulley *D*, and a loose pulley *C*. These pulleys should be of the same size with rounded faces. The arrangement of the pulleys must be in that of a square, as shown in the illustration, the side of which square is equivalent to the diameter of the pulleys at the centre of the faces. Put on to these an endless belt, and run in direction of the arrows, the loose pulleys *C* and *B* run in opposite directions to that of the shafts on which they turn. They carry a slack hold of the belt, and therefore are relieved of heavy strain on the shafts. If wide belts are used, the distance between the shafts ought to be ten times the width of the belt on the shafts. This arrangement is a successful method of transmitting power "around a corner" by a belt. There is no loss of contact of any part of the belt on the pulleys, and no lateral straining or tearing of the fibres, as is usually the case in quarter turn belts.

142. Fig. 37. Here from *A* on the main shaft the belt transmits

power to two shafts running at right angles to the main shaft, but all horizontal and in the same plane.

In FIG. 38, which is an elevation of the same arrangement of machinery, it will be observed that the driving face of the belt is changed between the pulleys *B* and *A*. The two flange guide pulleys are in this instance placed upon two separate shafts. They may be run, with the shafts fixed, or the shafts may be changed in position, but the arrangement as to centres of faces of pulleys must be the same as in FIG. 37. The general rule must be followed.

FIG. 39. Here is shown the usual method of transmitting power from one shaft to another, which shafts are placed at nearly right angles with each other, both being horizontal and in the same plane. Let *A* be the driving pulley and *E* the driven pulley. *B* and *C* are pulleys with flanges upon the lower edge, placed upon an upright shaft, the centre of the face of which is in exact line with the centre of the faces of the two pulleys *A* and *E*. Pulleys *B* and *C* are loose, running upon a shaft which is fixed. Belts will run either way, the two pulleys running in opposite directions.

143. Another method of transmitting power where shafts are situated at right angles to each other is shown in FIG. 40. In this case *A* is the driving pulley on a horizontal shaft, and *B* the driven pulley on an upright shaft. *C* is a tightener on a shaft parallel to the main shaft, with bearings arranged in a frame so that the shaft upon which *C* runs can be raised or lowered. Here a guide pulley *D* with a vertical shaft is run in fixed bearings. The course of the belt is indicated by the arrow.

The shafts may be at any angle with each other, and may not be in the same plane, the pulleys may differ much in diameter, and the belt may be crossed.

144. What cannot be done with the preceding methods of arranging the quarter-turn belt, may be done by the guide pulley devices shown in front and side elevation in FIG. 41, in which the vertical cylindrical staff, *A*, is secured to a flange, J, and held by a brace, *G*, to an over-head timber, *H*, or other fixture.

Upon this shaft are placed two hubs, *F*, held by set screws in any position, and each formed with a flat face, to which a flanged bracket, *C*, is bolted.

The upper bolt, *D*, is utilized as an axis about which the bracket can turn, and the lower bolt, *E*, working in a slot, permits the turning and holds the bracket at the inclination required by the belt.

In the centre of each flange, *C*, is secured a pin, *I*, upon which the pulleys, *B*, turn.

The facility with which these pulleys may have their axis inclined to accommodate the angle of a belt passing from a smaller to a larger pulley, from a higher to a lower shaft, and crossing to the opposite faces of pulleys, favors the employment of this combination, which has an all but universal adjustment, in many places where the previously described arrangements will not serve at all.

145. A mechanical engineer was required to manage a case of belting two shafts, set at right angles, one above the other, having pulleys of different diameters which were to run in opposite directions, and all of them lying close one to another. This was disposed of successfully by the use of mechanism exactly like that shown in FIG. 41, and this, with that shown in FIG. 39, are existing methods employed at the People's Works, Philadelphia, for belt driving around corners and in confined places.

FIG. 42 illustrates a mode of transmitting power by a flat belt directed by guide pulleys, from one shaft to another, in cases where the shafts are too near each other for direct driving. The shafts may or may not be parallel.

PLATES XII AND XIII.

Linkwork.

146. Four pairs of elements may be combined, and this combination may take place in different ways.

Suppose we have given four cylindrical pairs *ab*, *cd*, *ef*, and *gh*, Fig. 1, Plate 12.

Linkage. If we rigidly join each element of one pair to one element of another pair, we will have an endless chain, or *linkage*, returning on itself.

Link. The rigid body joining two elements of different pairs is called a *link*, a linkage being made up of a number of links.

147. Fig. 1, Plate 12, may be taken as an example of a simple linkage, which consists of four pairs, each being a cylindrical pin fitting a corresponding eye, the axes of the pins being parallel. Here each link has a motion in a circle relative to its adjacent link; but every motion of any link must, according to its connection, be accompanied by alterations in the positions of the remaining links. Hence, if we consider one link of the chain to be fixed, the motion of the remaining links may be referred to it, and their relative motion determined.

When in such a closed linkage we consider one of the links as fixed, we have a *mechanism*. Either link may be fixed, thus giving rise to as many mechanisms as the linkage has links. The fixed link forms a part of the frame of the machine, and may have a peculiar form.

148. The different parts of a linkage are named according to the motion they have in respect to the stationary link or frame.

Cranks, levers, and beams. Links which turn or oscillate are called *cranks*, *levers*, or *beams;* the term *crank* being usually applied to links making complete turns, as the crank of a steam engine.

Connecting rod; coupling rod. The rigid link connecting the oscillating or rotating links is called by various names, as *connecting rod*, *crank rod*, *eccentric rod*, *coupling rod*, *parallel rod*, etc., according to its location. It is attached to the pieces which it connects, by pins or ball and socket joints, and maintains the distance, between the centres of the pins or joints, invariable. Hence, it is called *the line of connection*, and the centres of the pins are called the connected points.

CRANK ARM. In a turning piece the perpendicular let fall from the turning point upon its axis of rotation is called the *arm* or *crank arm*.

149. ANGULAR VELOCITY RATIO. Before proceeding to the discussion of the different mechanisms that can be derived from the simple linkage shown in FIG. 3, we will determine the angular velocity ratios of the connected oscillating links. This may be found in two ways:

1°. By reference to an instantaneous axis.

Let $ABCD$ (FIG. 2, PLATE 12) represent the linkage, AB being fixed, AD and BC oscillating about points A and B respectively. Then O is the instantaneous axis about which CD is revolving at the given instant, as the points C and D are moving, for the instant, in lines Cc and Dd perpendicular to BC and AD respectively.

Produce CD to meet AB in E, and draw AS, BR, and OT perpendicular to CD.

By reference to the instantaneous axis, the linear velocities of C and D, for the instant, are proportional to OC and OD respectively, or

$$\frac{\text{Linear velocity of } D}{\text{Linear velocity of } C}=\frac{OD}{OC}$$

But the angular velocity of C is equal to its linear velocity divided by the radius BC; so we may write, remembering that the triangles OCT and BCR as well as ODT and ADS are similar,

$$\frac{\text{Ang. vel. of } AD}{\text{Ang. vel. of } BC}=\frac{OD}{AD}\div\frac{OC}{BC}=\frac{OT}{AS}\times\frac{BR}{OT}=\frac{BR}{AS}=\frac{BE}{AE}$$

2°. By the resolution of velocities.

Suppose the linkage to rock a very little; then C will move through the infinitely small space Cc (FIG. 2), while D will move through the correspondingly small space Dd.

Now the resolved motions of C and D along the line of connection CD must be equal; then

$$Dd \cos SDd = Cc \cos DCc;$$

or, since the angles BCR and ADS are complements of DCc and SDd respectively, we have

$$Dd \sin ADS = Cc \sin BCR$$

$$Dd \frac{AS}{AD} = Cc \frac{BR}{BC}.$$

Now putting for Cc and Dd their values in terms of the angles through which the links BC and AD respectively rotate, we have

$$\text{Ang. vel. of } AD \times AD \frac{AS}{AD} = \text{Ang. vel. of } BC \times BC \frac{BR}{BC}$$

$$\frac{\text{Ang. vel. of } AD}{\text{Ang. vel. of } BC} = \frac{BR}{AS} = \frac{BE}{AE}$$

Thus the angular velocities of the connected oscillating links are to each other *inversely* as the segments into which the centre line of the connecting link divides the line of centres.

This velocity ratio of course varies for every relative position of the links. When, however, the point T coincides with E, as in FIG. 3, PLATE 12, or when the points D and C are moving in lines parallel to each other, the angular velocity ratio is momentarily constant.

Having determined an expression for the velocity ratio in the simple linkage of FIG. 1, we will now discuss the various mechanisms that can be derived from it by fixing the different links.

150. CRANK AND LEVER.—Let the link D (FIG. 4, PLATE 12) be fixed, and suppose the crank A to revolve while the lever C oscillates about its axis d. In order that this may occur, we must always have the conditions:

1°, $A+B+C>D$. 3°, $A+B-C<D$.
2°, $A+D+C>B$. 4°, $B-A+C>D$.

1° and 2° must hold in order that any motion shall be possible; 3° can be seen from the triangle $a\,c_2\,d$, in the extreme right position $a\,b_2\,c_2\,d$, which cannot become a straight line; and 4° can be seen from the triangle $a\,c_1\,d$, in the left extreme position $a\,b_1\,c_1\,d$.

There are two positions c_1 and c_2 in the path of c at which the motion of the lever is reversed, and it will be noticed that if the lever C is the driver, it cannot, unaided, drive the crank A, as a pull or thrust on the rod D would only cause pressure on a, when c is either at c_1 or c_2. If A is the driver, this is not the case.

151. DEAD POINTS. The two points in the path of the crank at which it is impossible to start the mechanism by means of the connecting rod alone are called *dead points*.

The above form of linkage is applied in the beam engine as shown in FIG. 5, PLATE 12, the link D being formed by the engine frame; corresponding parts are lettered the same as in FIG. 4. The instantaneous axis is, for the position shown, at O, and for the instant the velocity of b is to the velocity of c as Ob is to Oc, or as bf is to ef; the line ef, drawn parallel to dc, is made use of when the point O comes beyond the limits of drawing.

The angle through which the lever C (FIG. 4) swings can be calculated for known values of A, B, C, and D.

From the triangle $a\,c_2\,d$

$$\cos adc_2 = \frac{C^2+D^2-(B+A)^2}{2\,CD}$$

and from the triangle $a\,c_1\,d$

$$\cos adc_1 = \frac{C^2+D^2-(B-A)^2}{2\,CD}$$

and $c_1dc_2 = adc_2 - adc_1$.

Thus the two angles adc_2 and adc_1 can be calculated, and their difference will give the angle required.

152. Given FIG. 4, PLATE 12, for the linkage $abcd$, a and b being fixed points about which the links A and B respectively revolve, $D = ad = 12$ inches, $A = ab = 3$ inches, $C = dc = 6$ inches, and $B = bc = 13.5$ inches. Through how great an angle does dc oscillate, while ab makes one complete revolution?

1st, to find the extreme positions of c, from a as a centre with a radius equal to $A+B$ describe an arc of a circle, and determine where this arc cuts the circumference of the circle described about a as a centre with a radius equal to the link C; this is one extreme position of the point c in its arc of oscillation about d. With a radius equal to $B-A$ determine a similar point of intersection with the same circle of radius C about d. This intersection determines the other extremity of the arc of oscillation of c about d. These points c_1 and c_2 limit the required arc of oscillation. The angle of oscillation is $c_1dc_2 = adc_2 - adc_1$.

From the figure we see that the following trigonometrical relations exist:

$$\cos adc_2 = \frac{\overline{ad}^2 + \overline{c_2d}^2 - \overline{ac_2}^2}{2ad \times c_2d}$$

$$\cos adc_2 = \frac{\overline{ad}^2 + \overline{c_2d}^2 - (A+B)^2}{2ad \times c_2d}$$

whence by substitution of given numerical values for AB, ad, and c_2d, we have

$$\cos adc_2 = \frac{144 + 36 - 272\tfrac{1}{4}}{144} = -.6406.$$

$$\cos adc_2 = -.6406, \quad adc_2 = -50^\circ\ 10'$$

$$\text{Similarly } \cos adc_1 = \frac{\overline{ad}^2 - \overline{c_1d}^2 - (B-A)^2}{2ad \times c_1d}$$

whence by substitution

$$\cos adc_1 = \frac{144 \times 36 - 110.25}{144} = .4844$$

$$\cos adc_1 = .4844, \quad adc_1 = 61^\circ +$$

$$\cos -(50^\circ\ 10') = \cos + (129^\circ\ 50')$$

$$\therefore\ adc_2 - adc_1 = 129^\circ\ 50' - 61^\circ = 68^\circ\ 50'$$

$$\therefore\ c_1dc_2 = 68^\circ\ 50'.$$

This outlines the method of determining the different parts of a 4-bar-linkage and the circumstances of motion of each part.

/ 153. DEAD POINTS. If at any instant the plane traversing or containing one of the crank arms, and its axis of rotation, coincides with the line of connection, or contains it also, the common perpendicular to the line of connection and the axis of that crank arm vanishes and the relations of the motions become indeterminate.

This position of the connected point of the crank arm in question at such instant is called a *dead point*. The velocity of the other connected point at this instant is zero.

The length of a stroke in a reciprocating piece is the distance between the two ends of the path in which a point of that line moves. When the point is connected by a link or connecting rod, with a point in a continuously rotating piece, the ends of the stroke of the reciprocating point correspond with the dead points of the continuously rotating piece. Hitherto the path of the reciprocating point has been taken as a straight line in direction, and passing through the centre of the crank circle, which is that point where the axis of rotation of the crank pierces the plane of the drawing. In this case the length of the stroke is twice the length of the crank arm, and the dead points are at opposite ends of that diameter of the crank circle which coincides with the line of direction of the path of the reciprocating point.

154. Now the problem is, where the crank arm and the line of connection are given in magnitude and the path of the connected point in the reciprocating piece is given in direction, to find the length of the stroke and the positions of the dead points.

If, as has been stated, the connected point in the reciprocating piece moves in a straight line, traversing and perpendicular to the axis of rotation of the turning piece, the length of the stroke is obviously twice the length of the crank arm, and the positions of the dead points are obviously at the ends of that diameter of the crank circle that coincides in direction with the path of the connected point of the reciprocating piece. If this connected point moves in any other path than that through the axis of rotation of the crank arm, let FF (FIG. 6, PLATE 12) be that path, and A the trace of the axis of rotation of the crank arm, or centre of the crank circle, and let $AD = AE$ be the crank arm, and let the circle $DEHG$ be the path of the connected point of the crank arm, or the path of the crank pin. Let AB be the line of connection minus the crank arm, then from A lay off AC, equal to the line of connection plus the crank arm. The length of this line of connection, or connecting rod, as it is usually called, and the length of the crank arm also being known, we can, as indicated, find the points B and C, and the length of the stroke BC.

1st, suppose we have given the direction FF, the position of A $\therefore Ax$ is known, the lengths of the crank AD and the connecting rod BD or EC, to find B, C, and BC. From A as a centre with a radius equal to the sum of the crank and connecting rod, describe an arc, its intersection with the line FF will determine the point *C*, which is one extremity of the stroke; similarly from *A* as a centre with a radius equal to the difference between the connecting rod and crank describe another arc, and its intersection with *FF* will determine *B*, the other extremity of the stroke; whence *BC*, the length of the stroke, may be determined graphically or analytically from the right angle triangles AxC and AxB, in which Ax, AC and AB are known, and Cx and Bx can be determined,

$$Cx - Bx = BC = \text{length of stroke.}$$

Now prolong *AB* and determine the dead point *D*; and *E*, the intersection of the crank circle and *AC*, is the other dead point. The angular distance between them can be determined analytically as can their position with reference to any required line in the diagram.

155. 2nd. Having given the position of the dead points and the centre of the crank circle and the lengths of the crank and connecting rod to determine the length and direction of the path of the connected point of the reciprocating piece, or to find the length and direction of the stroke.

Let *E* and *D* be the dead points, *A* the centre of the crank circle, and let *AD* and $DB = EC$ be the lengths of the crank and connecting rod respectively. From *D* through *A* draw *DB* equal in length to the given length of the connecting rod, and from *A* through *E* draw *AC* equal in length to the given lengths of the crank and connecting rod. Join the points *B* and *C*, thus determined, with a right line prolonged indefinitely, and thus determine the stroke *BC* in length, and *FF* in direction. When the path of the connected point in the reciprocating piece is in a straight line in direction, these processes can be expressed algebraically. Let S be the length of the stroke, L the length of the connecting rod and R the length of the crank arm. Then if the two ends of the stroke are in a straight line through the centre of the crank arm circle, we have $S = 2R$.

If the path is in any other direction, then S, $L - R$, and $L + R$ are the three sides of a triangle having the side S opposite the angle at the centre of the crank arm circle, so that, if θ be this angle, and equal to the supplement of the angular distance between the dead points, we will have,

$$S^2 = (L+R)^2 + (L-R)^2 - 2(L-R)(L+R)\cos\theta$$

$$\text{or } S^2 = 2(L^2+R^2) - 2(L^2-R^2)\cos\theta$$

$$\text{whence } \cos\theta = \frac{2L^2+2R^2-S^2}{2(L^2-R^2)}$$

156. The statement would be thus, given *FF*, the line of direction of the reciprocating point, two inches below *A*, the centre of the crank arm circle, the crank 6″, and the connecting rod 20″, to find the length of the stroke, and the dead points, or the required information might be to find the lengths of the crank and connecting rod, that shall give a reciprocating rectilinear motion of 8 inches to a sliding piece, the line of direction of the motion to pass 3 inches below the centre of the crank arm circle, and the reciprocating point when nearest this centre to be 30 inches from it, this 30 inches being measured on the line of direction of the sliding piece.

157. PARALLEL CRANKS. If in FIG. 4, PLATE 12, of the four-bar linkage, we make the opposite links equal, *i. e.*, $A = C$ and $B = D$, the crank chain becomes a parallelogram, as *ABCD* in the upper part of FIG. 7, PLATE 12. The lever *C* then becomes a crank equal to *A*, and (if *D* is fixed and some means of passing the dead points is provided) moves through the same angle as *A*. The nature of this mechanism is not changed by fixing any of the other links.

158. If, in the combination of two equal cranks, *A* and *C* (FIG. 7) with a connecting rod *B*, equal in length to the distance betweeen the centres of cranks, the crank *A* rotates right-handed until it comes over *D*, thus bringing the four links in line with each other, it will be noticed that in this position (one of the dead points) a further motion of *A* might cause *C* to turn either right-handed or left-handed. That is, for any given position of *A*, except the dead points, there are two possible positions of *C*, which can be found by drawing an arc of radius *B* about the extremity of *A* as centre, and noting where it cuts the path of the extremity of *C*. Thus a uniform right-handed rotation of *A* might cause a uniform right-handed rotation of *C*, or a variable left-handed rotation. To prevent this change of motion, and to insure the passage of the cranks by their dead points, two sets

of equal cranks may be combined, as shown in FIG. 7; the angle between the two sets of cranks being commonly taken 90°, so that when one set of cranks is at a dead point the other is in its best working position. Then a uniform rotation of the cranks A, A_1 will cause a unform rotation of C, C_1, thus giving a uniform and continuous velocity ratio between the axes of the two sets of cranks. Locomotives with coupled drivers are familiar examples of this arrangement.

159. FIG. 8, PLATE 12, shows another method of passing the dead points where a third equal crank B_1 is placed in the plane of the other two, and connected to them by links equal in length to the distance between its axis and the axes of the other cranks respectively.

160. Two cranks at right angles to each other, and located in one plane, could be connected with two others also at right angles, and located in another plane parallel to the first, by means of two parallel connecting rods sufficiently offset to enable them to clear each other in their motion, the distance between the two crank planes being made sufficient to admit of such an arrangement. This arrangement is practically of little value, especially when much force is to be transmitted, as offset rods, unless made very heavy, are likely to bend and cause binding of the crank pins.

161. It will be seen on reference to FIGS. 7 and 8, that the connecting rods B, B_1 and C, C_1 move in such a way that they are parallel to the line connecting the axes of the equal cranks, which carry them in all their positions, and also that all points in these rods move in circular paths of a radius equal to the length of the crank, *i. e.*, the rods may be said to have circular translation.

162. PARALLEL ROD. The term parallel rod or coupling rod is used to designate the rods such as B and B_1 (FIG. 7) employed to connect the driving axles of locomotives.

163. SLOW MOTION BY LINKWORK. The simple linkage shown in FIG. 4 can, if properly proportioned, be made to produce a slow motion of one of the cranks. Such a combination is shown in FIG. 9, PLATE 12, where two cranks A and C are arranged to turn on fixed centres and are connected by the link B. If the crank

A is turned right-handed, the crank C will also turn right-handed, but with decreasing velocity, which will become zero when the crank A reaches the position A_1 and is in line with the link B: any further motion of A will cause the link C to return toward its first position, its motion being slow at first and then gradually increasing. This mechanism has been applied in hand printing presses, where the crank C, attached to a screw, gave motion to the platen, and the crank A was furnished with a lever P so that it could conveniently be turned by the workman. When the combination is moved, as indicated by the arrow, at F, the crank C will have a decreasing motion, that of A being constant, causing the platen to descend with decreasing velocity, and therefore increasing force, until it reaches the type (the links then occupying the dotted position), when a very great pressure is obtained. See Art. 103, Goodeve's *Elements of Mechanism.*

164. In order that the greatest advantage may be gained, the crank C should be placed so that it will occupy a position perpendicular to the link B at the instant when A and B are in line, which is best shown in FIG. 10, PLATE 12, where the different links are represented by lines, and dc_1 is at right angles with b_1ac. To have this happen, the different links must be proportioned to satisfy the following equation:

$$bc - ab = \sqrt{ad^2 - cd^2}.$$

165. This mechanism is also employed in Corliss engines to operate the exhaust valves, which are attached to cranks like C (FIG. 9, PLATE 12), the crank A and link B being so proportioned that when the valves are at the positions of opening or closing, the crank C has its most rapid movement, while at the opened and closed positions it is nearly stationary, the crank A and link B being nearly in line with each other in one of these positions.

If in FIG. 10 we draw the line as, perpendicular to bc, and imagine a force F applied at the end of the lever ap, the pull along the link bc would be such that

$$F \times pa = \text{Pull on } bc \times as$$

$$\text{or Pull on } bc = \frac{F \times pa}{as}$$

166. The force necessary to balance the pull on *bc* would be supplied by the resistance to motion in the crank *cd*, and would be the pull felt at *c*. Now as the cranks turn, the connecting link *bc* approaches nearer and nearer to *a*, and *as* becomes smaller and smaller until it has no appreciable magnitude, and consequently the pull on the link *bc* increases enormously in the last instant of motion, In the position where $as = 0$, the crank *cd* should be at right angles, or nearly so, in order that the lever arm *dc* (that is, a perpendicular line drawn from the centre of motion to the line of the force) may be as long as possible. For example, if $F = 30$ pounds, $ap = 30''$, and the distance $as = \frac{1}{4}''$, we have

$$\text{Pull on } bc = \frac{30 \times 30}{\frac{1}{4}} = 3600 \text{ pounds.}$$

167. In the above combination (named after its inventor, Stanhope levers) the work is completed just as *ab* and *bc* come into line with each other, and no advantage would be obtained by continuing the motion; but if the links be properly proportioned, the crank *ab* can be made to make complete revolutions, as shown in FIG. 4, PLATE 12, and thus cause *ab* and *bc* to come into line twice in each revolution.

168. When this happens, we obtain in *ab* and *bc* a subordinate combination of links commonly known as *knuckle joint*, or *toggle joint*. This joint is much used where a gain of power is required, as in presses, machinery for punching and shearing iron, and in some forms of riveting machines.

169. TOGGLE JOINT. There are two forms of this joint corresponding to the positions ab_2c_2 and ab_1c_1 of the links *ab* and *bc* in FIG. 4, PLATE 12. FIG. 11, PLATE 12, shows the first case and FIG. 12, the second.

In the first form (FIG. 11) the links are generally made equal in length, but may be unequal, and are jointed together at *b*, the point *a* is fixed, and *c* is constrained to move in the line *ac*, thus exerting a pressure in the direction *ct*, when a force *F* is applied at the point *b* in the direction *bs*, perpendicular to *ac*. Here, as in FIG. 10, we have

$$\text{Thrust in } bc = \frac{F \times as}{ar}$$

The component of this thrust along *ac* can be found by resolving the force into two components, one *ct* along *ac*, and the other perpendicular to *ac*.

As the links *ab* and *bc* come more and more into line, the distance *ar* becomes smaller, the component of the thrust along *ac* approaches nearer and nearer to the thrust on *bc*, and when the links are in line, the thrust along *cb* is theoretically infinite, *ar* being then equal to zero.

170. In the second form (Fig. 12) the same reasoning applies as in the preceding case, and we have

$$\text{Pull on } ab = \frac{F \times cs}{cp}$$

which also becomes infinite when *ab* and *bc* come into line, thus making *cp* equal to zero.

171. Fig. 13, Plate 12, shows a metal shearing machine, in which a slow motion and consequently an advantage in power is obtained by means of linkwork. Here the long lever *A* is formed by a continuation of the crank *ab;* the crank or lever turning on *d* is connected to *ab* by the link *bc*, *a* and *d* being fixed centres carried by the frame of the machine, which forms the fourth link of the chain. The metal to be sheared is placed at *S*, and the power is applied at the end of the lever *A*: The operation of the machine can be easily understood from the figure. The links *ab* and *bc*, *c* moving nearly in a straight line, form a *toggle joint.*

172. Double oscillation by linkwork. If we combine the links *dc*, *ce*, and *ie* as show in Fig. 14, Plate 12, *d* and *i* being fixed centres, with a crank *ab* (turning on the fixed centre *a*), and a connecting rod *bc*, the lever *ie* can be made to make a double oscillation during one revolution of the crank *ab*. In order that this may properly occur, the extreme positions of the point *c* should be equidistant from the centre line *de*, and the extreme positions of *e* should be equidistant from the perpendicular dropped from the centre *i* on the line *de*. If we continue the link *ie* to *f*, and connect it with the links *fg* and *gh*, *h* being a fixed centre, and so chosen that the extreme positions of *g* are equidistant from the perpendicular dropped from *h* upon *ig*, the lever *hm* will make four oscillations for each revolution of the the crank *ab*.

173. Returning again to the four-bar linkage of FIG. 4, we can substitute for the lever *C* a small sector of an annular cylinder, and enclose this in a circular slot (FIG. 15) rigidly connected with the centre about which the crank *A* revolves. If the centre line of the slot and the centre of the sector *C* be placed at a distance from *d* equal to the distance *dc* in FIG. 4, the sector has exactly the same relative motions as it would have had, had it been connected to the lever *C*. The elementary links *Ad* and *Cd* (FIG. 15, PLATE 12) take the place of the links *C* and *D* of FIG. 4, PLATE 12. We can now without introducing any constructive difficulties make the radius of the slot of any required magnitude, the slot and the sliding piece approaching more nearly a straight line than before. From making the radius infinite it necessarily follows that *cd* and *ad* (FIG. 4, PLATE 12) or *Cd* and *Ad*, FIG. 15, both become infinite, and we have therefore the mechanism shown in FIG. 16, which might be called the *sliding block linkage*.

174. This is a common form of linkage, known as the crank and connecting rod, and its object is to convert circular into reciprocating straight line motion, and the reverse. As an example of the first case we have a common pump; of the second case the steam engine where the link *A* (FIG. 16, PLATE 12) is the crank, *B* the connecting rod, *D* the frame, and the sliding piece *C* the crosshead. To *C* is attached the piston rod, which communicates to *C* a reciprocating straight line motion, due to the alternate action of the steam in the cylinder on each side of the piston; this reciprocating motion of *C* is converted into the circular motion of rotation of the shaft *S* by the crank and connecting rod here shown. In the first case, the shaft *S* would be the driver, and its circular motion would be converted into the reciprocating motion of *C* by the same crank and connecting rod; the plunger or piston rod of the pump to be worked, would then be attached to *C*. By successively fixing the various parts of the mechanism shown in FIG. 16, PLATE 12, various forms of mechanism arise. If we consider the link *B* as fixed, we obtain the *swinging block linkage*, shown in FIG. 17, PLATE 12.

This is the form used in oscillating engines where *A* represents the crank, *D* the piston rod, and *C* the cylinder, which fills the place of swinging block. The relative motion of *C* and *D* is the same whether D slides through C as a guide, or slides on C as a pin or stud.

175. CONNECTING ROD AND CRANK. Considering the link D (FIG. 16, PLATE 12) as fixed, we obtain the mechanism commonly used in pumps and direct acting steam engines. When employed in a steam engine, the block C is the driver and the crank A the follower; in a pump the reverse is the case.

MOVEMENT OF CROSSHEAD. In FIG. 1, PLATE 13, let ab represent the crank, bc the connecting rod, and mn the path of the point c in the crosshead. The travel of the crosshead mn is equal to twice the length of the crank and its distance from a varies between $B+A=an$ and $B-A=am$, A being the length of the crank, and B the length of the connecting rod.

To find the distance the crosshead c has moved from n, the beginning of its stroke or travel, let the angle made by the crank with the line an be represented by θ, and draw bg perpendicular to an. The movement of the crosshead from the beginning of its stroke is, for the angular motion θ of the crank,

$$cn = an - ac = an - (ag + gc).$$

From the right triangle bcg,

$$gc = \sqrt{bc^2 - bg^2}.$$

From the right triangle agb,

$$ag = ab \cos\theta \quad \text{and} \quad bg = ab \sin\theta$$

$$\begin{aligned} \text{hence} \quad cn &= an - ab\cos\theta - \sqrt{bc^2 - ab^2 \sin^2\theta} \\ &= A + B - A\cos\theta - \sqrt{B^2 - A^2\sin^2\theta} \qquad \text{(A)} \\ &= A(1-\cos\theta) + B\left(1 - \sqrt{1 - \frac{A^2}{B^2}\sin^2\theta}\right) \end{aligned}$$

If the length of the connecting rod has a certain relation to that of the crank, A being the length of the crank, and lA that of the rod, we have, substituting lA for B in equation (A),

$$cn = A(1 + l - \cos\theta - \sqrt{l^2 - \sin^2\theta}) \quad . \quad . \quad (1)$$

176. VELOCITY RATIO. In FIG. 1, PLATE 13, continue the line of the connecting rod to s, and draw the line as through a and perpendicular to am. The instantaneous centre of the rod bc is at O, found by drawing the lines bO and cO perpendicular

to the lines of motion for the instant of the points b and c respectively.

As the velocities of the points b and c are proportional to their distances from the instantaneous centre O, we have,

$$\frac{\text{Velocity of } c}{\text{Velocity of } b}=\frac{Oc}{Ob}=\frac{as}{ab}=\frac{as}{A} \qquad \text{(B)}$$

as the triangles abs and bcO are similar.

From the similar triangles cas and cbg we have

$$\frac{as}{bg}=\frac{ac}{gc}, \text{ or } as=bg\frac{ac}{gc}=bg\frac{ag+gc}{gc}$$

$$as=\frac{A\sin\theta(A\cos\theta+\sqrt{B^2-A^2\sin^2\theta})}{\sqrt{B^2-A^2\sin^2\theta}}$$

Substituting in (B), we have

$$\frac{\text{Velocity of } c}{\text{Velocity of } b}=\frac{as}{A}=\sin\theta+\frac{A\sin\theta\cos\theta}{\sqrt{B^2-A^2\sin^2\theta}} \qquad (2)$$ ✓

The velocity of b being constant, that of c can be found by equation (2).

If the length of the link B be made infinite, and the crank A rotate uniformly then the guided point c will have a simple harmonic motion.

177. In the crank and connecting rod mechanism shown in FIG. 1, PLATE 13, the length of the crank A is 10″, and the length of the connecting rod B is 60″. How far will the crosshead move from the beginning of its stroke, when the crank has moved through an angle of 60° measured from the line of direction of the reciprocating motion?

In the figure cn is the required distance, and to find it we have $ab=A=10$ inches, $bc=B=60$ inches, and the angle $bac=\theta=60°$.

From an inspection of the figure we see that $cn=an-ac$, and $ac=ag+gc$, now $ag=ab\cos\theta$, and $gc=\sqrt{bc^2-bg^2}$, since bg is drawn perpendicular to ac.

Again, $bg=ab\sin bac=A\sin\theta$, as n is the extreme position of the reciprocating point, $an=A+B$, since the crank and connecting rod are in line, or at a dead point, when the reciprocating point is at either end of its path, now substituting these values in the expression $cn=an-ac$, we have the expression,

$$cn = A + B - A\cos\theta - \sqrt{B^2 - A^2\sin^2\theta}$$

$$\text{or } cn = A(1 - \cos\theta) + B - \sqrt{B^2 - A^2\sin^2\theta}$$

as per given data $B = 6A$, $\cos\theta = \frac{1}{2}$; $\sin\theta = \frac{1}{2}\sqrt{3}$.

$$cn = A(6 + 1 - \tfrac{1}{2} - \sqrt{36 - \tfrac{1}{4}\times 3})$$

$$cn = 5.63. \text{ Answer.}$$

That is, when the crank has made $\frac{1}{8}$ of its travel, the reciprocating point has only advanced 5.63″, when a proportional travel for the reciprocating point would have been $\frac{20}{3} = 6.66''$.

2nd. What will be the crank angle when the crosshead is in the middle of the stroke? *i. e.*, required the value of θ when $cn = 10$ inches.

From the preceding case we have

$$cn = 10 = (6 + 1 - \cos\theta - \sqrt{36 - \sin^2\theta})$$

$$\text{or } 6 - \cos\theta = \sqrt{36 - \sin^2\theta}$$

squaring both members we have

$$36 - 12\cos\theta + \cos^2\theta = 36 - \sin^2\theta$$

$$\cos^2\theta + \sin^2\theta = 12\cos\theta$$

$$\cos\theta = \tfrac{1}{12} = .08343,$$

$$\theta = 85^\circ 13' 20''.$$

From which we discover that the crosshead completes the half of its travel before the crank has completed 90° or the half of its travel.

178. By harmonic motion is to be understood the motion of a point which moves to and fro in a straight line in such a manner that its velocity at every instant is equal to the component, parallel to that straight line, of the velocity of another point which revolves uniformly in a circle. The length of this straight line referred to, is called the travel of the reciprocating point and is equal to the diameter of that circle, which is the path of the revolving point. Harmonic motion is exactly realized by any point in a slotted crosshead sliding rod, driven by a uniformly rotating crank, that is, in the mechanism, known as the crank and infinite connecting rod.

The angle which the crank makes with the line of the dead point is called the *phase of the motion*. The velocity of the reciprocating point varies as the sine of the phase, and the distance of the reciprocating point from its middle position varies as the cosine of the phase.

179. Harmonic motion is approximately realized by any point in a piece, such as a piston driven by means of a connecting rod and a uniformly rotating crank. The extent of error in that approximation may be expressed either in the form of greatest error in position or of greatest error in velocity. The greatest error in position is the distance of the reciprocating point from the middle of its travel, when the crank is midway between its dead points. When the line of stroke of the reciprocating piece passes through the axis of the crank, this greatest error in position may be expressed algebraically by the expression

$$l-\sqrt{l^2-c^2},$$

in which c and l denote the length of the crank and connecting rod respectively.

When the line of stroke of the reciprocating piece does not pass through the axis of rotation of the crank, there are other errors in the approximation to true harmonic motion, arising from the two dead points not being diametrically opposite. These errors may be found in the manner indicated in the discussion on dead points.

If in Fig. 2, Plate 13, we draw tangents to the path of the crank pin, from the centre of motion of C, they will divide the circle (by their points of tangency) into two parts, and, if the crank A has a uniform rotation, the times of oscillation of the slotted link C will be proportional to the two parts of the crank-pin circle.

This mechanism is very generally used in shaping and slotting machines of English manufacture, where the slotted link C is connected to the toolholder M by means of a link, the tool being arranged to take its cut while the crank is moving over the large sector, and to return while the crank is moving over the smaller sector of the crank-pin circle. Thus a slow mean cutting motion is obtained, and a quick mean return of the tool, the times of the motions being proportional to the arcs of the crank-pin circle between the two extreme positions of the link C.

181. WHITWORTH QUICK RETURN MOTION. If we consider the link A (FIG. 16, PLATE 12) as fixed, we will obtain the mechanism shown in FIG. 3, PLATE 13, which might be called the "*turning block linkage*," which has been used by Whitworth and others as a quick return motion. Here B becomes a crank, and, if it has a uniform rotation, will drive D by means of the block C with a varying angular velocity. This mechanism somewhat resembles that of FIG. 2, but here the centre of oscillation of C falls within the crank-pin circle instead of without, as in FIG. 2.

182. ANGULAR VELOCITY RATIO. In FIG. 4, PLATE 13, let ab be the driving crank, and let pd be the following slotted crank. When the crank ab is in the position ah, pd will be in line with it. If now the crank turns right-handed until b reaches e, pd will have passed through the angle edh, less than the crank angle eah by the angle aed; thus ab has gained on pd an angle dea. Now, if the motion is continued to ag, the cranks are then again in line, and the slotted crank pd has gained the angle previously lost; thus ab alternately gains and falls behind pd. In the next one-half revolution ab falls behind pd, and then gains the angle lost when b reaches h. While pd makes the half-revolution above the line ef, the crank pin b moves through the arc egf, and during the remaining half revolution of pd, b moves through the arc fhe.

To find the angular velocity ratio, draw the line of connection between ab and pd; *i. e.*, br perpendicular to pd, and note the point r where it cuts the line of centres; then

$$\frac{\text{Angular velocity of } pd}{\text{Angular velocity of } ab} = \frac{ar}{dr}.$$

See FIGS. 2 and 15, PLATE 12, of which the mechanism is a particular case.

183. The angular velocities would also be inversely proportional to the perpendiculars let fall from the axes of the cranks upon the line of connection.

If we assume the angular velocity of ab as constant and equal to unity, that of pd will be greatest when b is at $g = \frac{ag}{dg}$ and least when *b* is at $h = \frac{ah}{dh}$. When *b* occupies either of the positions *e* or *f*, the angular velocities are equal, as in these positions the perpendiculars dropped from the axes upon the line of connection are equal.

184. If now we suppose the slotted crank *dp* to actuate a tool holder moving in the line *eft* (drawn through *d* perpendicular to the line of centres *ad*) by means of a connecting rod *pt*, a point on the holder, as *t*, will, on a left-handed uniform revolution of *ab*, make its stroke outward from *d* while the crank pin *b* travels through the arc *ehf;* and the return of *t* will be accomplished while the crank pin moves through the arc *fge*. Thus the time of the outward stroke is to the time of the return stroke as the arc *ehf* is to the arc *fge*.

If it is desired that the forward or outward stroke shall occupy two-thirds of the revolution of the driving crank *ab*, and the return stroke the remaining one-third revolution, it is only necessary to so locate the perpendicular *edf* that it will divide the crank-pin circle into two parts, *ehf* and *fge*, in the proportion of two to one. The point *d* where this perpendicular cuts the line of centres is the position of the centre of the slotted crank. By so arranging the slotted crank that the point *p* for the attachment of the connecting rod is adjustable on *pd*, the stroke of the point *t* can be changed. If the point of attachment *t*, of the rod to the tool holder is also made adjustable, the position of the stroke as well as its length can be changed.

185. FIG. 5, PLATE 13, shows this mechanism as practically constructed. The spur wheel *B*, actuated by a pinion (not shown), has a uniform left-handed rotation on the large pin *A* (which is a part of the frame of the machine), and carries a pin turning in the block *C*, which slides in a radial groove of the piece *D*. This piece *D* turns upon a pin placed out of the centre of *A*, and is provided with a T slot for the attachment of a connecting rod as shown opposite to the slot for the block *C*. The line of motion is drawn through the axis of the piece *D* and perpendicular to the line of centres of *B* and *D*, which correspond respectively to the pieces *ab* and *dp* in FIG. 4. This mechanism is very much

used in shaping and slotting machines of english manufacture to obtain a slow mean cutting motion for the tool combined with a quick mean return.

186. By fixing the block *C*, we obtain the mechanism shown in FIG. 6, PLATE 13. The connecting rod *B* now swings about a fixed axis in *C*, and the slide *D* moves rectilinearly to and fro in the block *C*, which is now the frame; the crank *A*, now a connecting rod, has a complex motion made up of a combined oscillation and rotation.

187. THE ISOSCELES SLIDING BLOCK LINKAGE. If we make the length of the connecting rod *B* (FIG. 6, PLATE 13), equal to that of the crank *A*, we will obtain the linkage *abc* (FIG. 7). Here, if we consider *ab* as the driver, and *c* to start from the position c'' at the right, it will be found that when the crank *ab* is at an angle of 90° with ac'', the path of *c*, in the block *c* is directly over *a*, and any further rotation of *ab* will only cause a similar rotation of *bc*.

In order to cause *c* to continue in its path when *ab* reaches the 90° position and coincides with *ec*, some such contrivance as is shown at e' and e'' is necessary to prevent the point *e* from revolving about *a* as a centre in place of reciprocating on the straight line $e'e''$. The travel of *c* equals 4 times the length of *ab*. This is an important form of mechanism and is used in practice.

188. To show graphically how to construct a swinging block linkage, such that the periods of advance and return are to each as 3 to 2; in FIG. 8, PLATE 13, let *xyz* be the circular path of the crank pin of the crank *AS*. Divide this circumference into 5 equal parts, marked 1, 2, 3, 4, 5, respectively, at the points of division. At the points of division marked 2 and 5, draw the tangents to the crank-pin circle, the intersection, *O*, of these two tangents, will be the centre of the pivot of the swinging block.

Let the crank revolve uniformly right-handed, and suppose an arm carrying a reciprocating piece *MN* to be attached at *F*. Then the period of the motion of *MN*, right-handed, will be to the period of its motion left-handed, as the arc 2, 3, 4, 5, is to the arc 5, 1, 2, or as 3 to 2.

If now a cutting tool be attached to the right-hand end of the reciprocating piece, it will have a period of advance motion that will be to its period of return motion as 3 to 2. That is, it will have a quick return motion $\frac{3}{2}$ as fast as its advance motion.

189. To show how to construct a Whitworth quick return motion so that the period of return shall equal ½ the period of advance: in Fig. 9, Plate 13, let *ab* be the crank and in this case the driver, and *rdh* the line of centres; about *a* as a centre describe the circle which will represent the travel of the crank pin; divide this circle into three equal parts, and through *e* and *f*, adjacent points of division, draw *ef*, perpendicular to the line of centres, *rdh*, thus making the arc *egf* ⅓ of the circumference of the crank-pin circle and ½ of the remaining arc *ehf;* from *d*, the intersection of *ef* and *rdh*, draw *dp* and make it any suitable length. It is then the axis of a slotted arm in which works a roller or stud, *b*, at the end of the crank *ab*, which roller or stud is free to move therein, so that a rotation of the driver *ab* about *a* as a centre will produce a rotation of *dp* about *d* as a centre, in such a manner, that the period of rotation while *dp* is above *ef* is ½ the period of rotation while it is below *ef*. Then if *pd* be the crank, *pt* the connecting rod, and *T* the reciprocating piece, constrained to move in the line of direction *xy* passing through *d*, the centre of rotation of the crank *pd*, and if a punch be attached to *T*, and the system be caused to rotate left-handed, we will have a quick return motion, whose period of advance is twice its period of return. From the inspection of the figure it appears that the period of motion of *T* to the right, is to its period of motion in opposite direction, as the arc *ehf* is to the arc *egf*, or as by construction as 2 to 1.

The line of action, the degree of motion, and the length of the connecting rod are, or can be made, adjustable.

190. The object of a quick return motion is apparent, being an effort to economise time when there is no useful work to be done and thus to reduce to a minimum the necessary losses in machinery. If the parts as determined are arranged as in Fig. 5, we have the mechanism as it occurs. The pin in the block *C*, and the centre of the gear wheel *B*, or the line *ab*, correspond to the crank, as this pin is fast to the wheel *B;* the groove in *D* corresponds to the line *pd*, and the end of the crank being free

to move in this groove, the combination is completed by the attachment of the connecting rod. This is adjustable by means of the T slot and set screw in the sliding block *E*, which block is of T-shaped cross section to fit the slot.

✓ 191. OLDHAM'S COUPLING. The mechanism shown in FIG. 10, PLATE 13, known as *Oldham's Coupling*, is an interesting example of mechanism. Its object is to connect two parallel shafts placed a short distance apart so as to communicate a uniform rotation from one to the other.

The bearings for the two shafts *a* and *b* are in piece *A*; the parts *B*, *C*, and *D* are drawn separated, at the right of the figure, to make their construction clearer. The piece *C* has two diametrical slides *c* and *d* placed on its opposite sides and at right angles to each other. The grooves *c* and *d*, in the pieces *B* and *D* respectively, fit the corresponding slides similarly lettered on *C*.

192. Let us consider the mechanism shown in FIG. 11, PLATE 13, and commonly known as the *elliptic trammel*, employed to trace ellipses.

Here the piece *C* is provided with two straight grooves at right angles to each other in which the blocks *a* and *b* slide, the blocks being made long enough to pass over the intersection of the grooves at *o*. Each block is provided with a circular hole into which the adjustable pins *a* and *b* on the rod *ap* are fitted, the pins *a* and *b* being enlarged at the top and provided with cylindrical holes through which the rod *ap* can slide, thumb screws serving to hold the pins in position on the rod when adjusted. If now a pencil *p* is placed at the end of the rod, and the rod is then rotated, it will trace an ellipse upon the plane of the piece *C*, the blocks travelling back and forth in the grooves and always keeping a distance *ab* apart. The semi-major axis of the traced ellipse is *ap*, and the semi-minor axis is *bp*.

In FIG. 11 drop a perpendicular from *p* on the line *BB*, and mark the point where it intersects the line *n*; similarly drop a perpendicular from *p* on the line *DD*, and mark its intersection with *DD*, *m*.

Let $pm = on = x$, $pa = a$

$om = pn = y$, $pb = b$

$$\text{Then } \frac{x}{a} = \frac{pm}{pa} \text{ and } \frac{y}{b} = \frac{pn}{pb} = \frac{am}{pa}$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{\overline{pm}^2 + \overline{am}^2}{\overline{pa}^2} = 1$$

$$\text{Since } \overline{pm}^2 + \overline{am}^2 = \overline{pa}^2,$$

which is the equation of an ellipse referred to its centre as the origin.

Now in drawing an ellipse, the paper is fixed, and the pencil is moved over it, but in turning an ellipse in a lathe, the tool, which has the same position as the pencil, is fixed, and the piece to be turned should have such a motion as would compel the tool to cut the ellipse. See models of elliptic trammel and elliptic chuck.

193. To show how to construct an elliptic trammel to describe an ellipse whose major axis shall be 2½ times its minor axis; in FIG. 12, PLATE 13, draw fk and ad at right angles to each other, make $fk = 3$ inches and $ad = 7.5$ inches, then these are the two axes. Construct two slotted arms QQ and MN, with blocks A and B free to move in the slots and long enough to pass the intersection of the slots at O, and thus ever keep in their own path. Fasten a rigid arm CS to each block with a pivot at A and B, in such a manner that $AC = 3.75$ inches, $BC = 1.5$ inches and $AB = 2.25$ inches; then $AB =$ the difference between the semi axes. If a pencil be attached at C, and the rod CS be caused to rotate, or the slides A and B be caused to reciprocate in their slotted ways, the pencil C will describe an ellipse, whose semi-major axis is AC and whose semi-minor axis is BC, for, taking any position, as the one in the drawing. We draw CP perpendicular to ad and CH perpendicular to fk.

Call $AC = a$, $BC = b$,

$CP = y$, $CH = x$.

Then for any point as C

$$\frac{x}{a} = \frac{PO}{CA} = \frac{CH}{CA}$$

$$\text{and } \frac{y}{b} = \frac{CP}{BC} = \frac{HA}{AC}$$

$$\therefore \frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{\overline{CH}^2}{\overline{CA}^2} + \frac{\overline{HA}^2}{\overline{CA}^2} = \frac{\overline{CH}^2 + \overline{HA}^2}{\overline{AC}^2}$$

$$\text{but } \overline{CH}^2 + \overline{AH}^2 = \overline{AC}^2$$

$$\therefore \frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{\overline{AC}^2}{\overline{AC}^2} = 1$$

which is the equation of an ellipse ∴ the curve described by the pencil at the point *C*, the rod *CS* moving as described, is an ellipse; since *C* is any point of such a curve, the demonstration holds true for every point, since every point is similarly generated.

PLATES XIV AND XV.

Parallel Motions.

194. A parallel motion in mechanism is a combination of turning pieces, usually links and levers, designed to guide the motion of a reciprocating piece, either exactly, or approximately in a straight line, so as to avoid the friction that necessarily arises from the use of straight guides, for accomplishing the same purpose. The most common application of parallel motions, in practice, is in the attachment of pump or piston rods to the common lever that acts as the driver for these rods.

195. Some parallel motions are exact, that is, they guide the pump or piston rod, or valve rod or other reciprocating piece, in an exact straight line; but these parallel motions cannot always be conveniently made use of. Other parallel motions are only approximate, that is, the path of the reciprocating piece to which they act as guides, is near enough to a straight line to accomplish the practical object in view; these are the most frequent forms of parallel motions met with in practice.

196. They are usually designed upon the principle, that the two extreme positions, and the middle position of the guided point shall be exactly in a straight line, care being taken at the same time that the deviations of the intermediate parts of the path of that point from that straight line shall be as small as possible. Often the path of the guided point is the arc of a curve or circle, such that, for the extent of the reciprocation the curve and the straight line are practically coincident. There are purposes for which no merely approximate parallel motion is sufficiently accurate, such as the guiding of the tool in a planing machine whose motion ought to be absolutely rectilinear. The principle of the Hypocycloid tells us when one circle rolls on the interior of the circumference of a circle of exactly twice its radius, any point of the first circle will trace an exact straight line, which line is a diameter of the second circle. This suggests a linkage mechanism that will accomplish an exact straight line motion.

This principle has been used, as an exact straight line motion for a piston rod (see FIG. 1, PLATE 14), the head of the piston rod *FF* being jointed to a pin *P* on the perimeter of the inner wheel *H* and the crank *C* to another pin *O* at the centre of that wheel *H*. The crank being the driver, its rotation on the shaft *S* will cause this wheel *H* to move as on the interior of an imaginary wheel *n n n*, whose radius *PQ* equals the diameter *PE* of the inner wheel *H* or twice the length of the crank-arm *C*, in this manner the piston head takes a constrained rectilinear reciprocating motion coinciding in direction with *AB*, the line of direction of the diameter of the imaginary circle, *n n n*, and equalling it in length.

This is by no means a common straight line motion for piston rods. It is used in a small model of hand drill where the rotation is affected by a hand crank and lever and toothed wheels. The mechanical objections are evident, as without guides, when *PQ* coincides with *PB*, the motion becomes indeterminate; it is an application of FIG. 7, PLATE 13. It is noticed here as an interesting mechanical combination. (See Rankine's *Machinery and Mill Work*, Art. 250.

197. PEAUCELLIER'S STRAIGHT-LINE MOTION. FIG. 2, PLATE 14, shows a linkage invented by M. Peaucellier, for describing an exact straight line within the limits of its motion.

It consists of eight links jointed at their ends. Four of these links, *AB*, *AP*, *A'B*, and *A'P*, are equal to each other and form a cell; the two equal links *AD* and *A'D* connect the opposite points *A* and *A'* of the cell with the fixed centre of motion *D*; the link $BC =$ one-half *BD*, and oscillates on the fixed centre *C*; *CD* thus forming the fixed link equal in length to *BC*.

If now the system be moved within the limits possible by its construction (that is, until the links *AB* and *BC*, and *A'B* and *BC* come into line on opposite sides of the centre line of motion *CD*), the cell will open out and close, the points *A* and *A'* will describe circular arcs about *D*, and *B* about *C*. The latter, if completed, would pass through *D*, as shown in FIG. 3, PLATE 14. Finally the point *P* will describe a straight line *SS* perpendicular to the line of centres *CD* and passing through the point *P*, the central position of the describing point. For proof see Art. 109, Schwamb.

198. Fig. 4, Plate 14, represents another exact parallel or straight-line motion, known as *Scott Russell's* parallel motion and it appears in various forms.

The arm *CD* turns on the axis *C* and is jointed at *D* to the middle of the bar *ADB*, whose length is double that of *CD*, and one of whose ends *B* is jointed to a sliding piece, free to move between the guides *G G* in the straight line *BC*.

Draw *BE* perpendicular to *BC* and cutting *CD* produced at *E*, then *E* is the instantaneous axis of the bar *ADB* and the direction of the motion of *A* is, at every instant of such motion, perpendicular to *EA*; that is, *A* moves along the straight line *ACa*. The stroke of *A* is *ACa* extending to equal distances on either side of *C*, and equal to twice the chord of the arc *dD*, while the stroke of *B* is only equal to twice the deflection of the arc *dD* from a straight line through *d* and *D*, and thus *A* is guided, through a comparatively long stroke by the sliding of *B* through a comparatively short stroke, and by the rotatory motions at the points *C*, *D*, and *B*, about these same points as centres.

199. Another proof of the Scott-Russell straight-line motion is based on the fact, that as $AD = BD = CD$, the points *A*, *C*, and *B* are ever on the circumference of a circle about *D*, also *ADB* is a straight line, and must therefore be the diameter of the circle, whence the angle *ACB*, being inscribed in a semi-circle is a right angle, and as this is true for every position, the line *AC* must always be perpendicular to *BC*, and the path of the guided point, must be a right line and at right angles to *BC*.

In practice, this and other linkages used to guide a point are often made in pairs and located on both sides of the guided point, which is placed midway between them.

200. Approximate straight line motions somewhat resembling the preceding may be obtained by guiding the links entirely by oscillating links, instead of by a link and slide.

1°. In the link *CP* (Fig. 5, Plate 14) choose a convenient point *S* whose mean position is S_1, and whose extreme positions are *S* and S_2. Through these three points pass a circular arc SS_1S_2, the centre of which will be found on the line *AC*. Join *S* and *T* by a link, *ST*, and the two links *AB* and *ST* will so guide *PS*, that the mean and extreme positions of P will be found on the line PP_2.

2°. The point C may be made to move very nearly in a straight line CC_1 by means of a link CD centered on a perpendicular erected at the middle point of the path of C. The longer this link the nearer the path of C will approach a straight line.

201. This form of straight-line motion has been applied in a form of small stationary engines, commonly known as grasshopper engines, where CBP (FIG. 5), extended beyond P, forms the beam of the engine, its right hand end being supported by the link CD. The piston rod is attached, by means of a crosshead, to the point P, which describes a straight line, and the connecting rod is attached to a point in the line CP produced, as P', FIG. 6, both piston rod and connecting rod passing downward from CP. In this case it will be noticed that the pressure on the fulcrum C, of the beam, is equal to the difference of the pressures on the crosshead-pin and the crank-pin instead of the sum, as in the ordinary form of beam engine.

In this second form of motion it is not always convenient to place the point A in the line of motion PP_2, and it is often located on one side, as shown in FIG. 6.

202. WATT'S PARALLEL MOTION. This motion, the one most commonly used, was invented by James Watt in the year 1784; it is an application of the modified form of the *double rocking lever.* FIG, 7, PLATE 14, shows such a motion; here the two links or levers DA and BC, connected by the link BA, oscillate on the fixed centres D and C, and any point, as P in the connecting link BA, will describe a complex curve. If the point P be properly chosen, a double looped curve *Padbc* will be obtained, two parts of which, *ab* and *cd*, are nearly straight lines. In designing such a motion it is customary to use only a portion *eF* of one of the approximate straight lines, and to so proportion the different links, that the extreme and middle points *e*, *F*, and P shall be on a line perpendicular to the centre lines of the levers DA and BC in their middle positions, when they should be taken parallel to each other.

203. In FIG. 8, PLATE 14, the linkage is shown in its mid-position at $DABC$ and in the extreme positions $DA_1B'C$ and DA_2B_2C.

The centres D and C are so chosen that the middle points m and n of the versed sines Av and Bu are found on the line SS, drawn perpendicular to the links DA and BC in their middle position, when they are taken parallel. The guided point P is found at the intersection of AB and this perpendicular.

To show that the guided point P is on the line SS in its extreme positions, draw the lines A_1r and $B't$ parallel to AD and BC; then by the construction $A_1r = mv = Am$, and $B't = nu = Bn$ and $AB = A_1B'$; therefore the figures $mABn$ and $rA_1B't$ are equal, and P_1 is on the line SS. In the same way, P_2 may be shown to be on the line SS.

From the similar triangles AmP and BnP we have

$$PA : PB = Am : Bn$$

also from the parallelogram $A_1B'B_2A_2$, crossed by the line SS we have

$$A_1A_2 = B'B_2 = P_1P_2 = s = \text{the stroke.}$$

204. The general construction of the ordinary form of Watt's approximate parallel motion is shown in FIG. 9, PLATE 14. Ac is one arm of the walking beam of the engine, turning about an axis at the main centre c. AB is the main link, connecting the end A of the walking beam, with the piston rod BD. Tt is the back link, equal and parallel to the main link AB; and BT is the parallel bar, equal and parallel to the part At of the walking beam, and completing the parallelogram $AtTB$. The piston rod head, B, is to be guided so that its highest, middle, and lowest positions, shall be in one straight line, and this is effected by guiding in the same manner the point P, where the straight line Bc cuts the back link.

The guiding of the point P is effected by means of the radius bar or bridle CT, which is a lever that turns about an axis at C, and is jointed at T to the back link.

The linkage $CTtc$ is the Watt's parallel motion and the point P is the guided point. From the point P a pump rod is often hung.

205. FIG. 10, PLATE 14, shows on a larger scale, the principle according to which the point P is guided. Ss is the line of stroke of the point P; P_1, P_2, and P_3 are the upper, middle and lower

posititions of the guided point; $T_1P_1t_1$, $T_2P_2t_2$, and $T_3P_3t_3$ are the corresponding positions of the link; and cT_1, CT_2, and CT_3, ct_1, ct_2, and ct_3, are the corresponding positions of the levers, which guide that link. In designing a parallel motion, the principal problem is to adjust the relative positions and proportions of the levers and link, so that the three positions of the guided point, P_1, P_2, P_3, shall lie in the straight line Ss. There are also subordinate problems, the first of which has for its object to make the deviation of the guided point from the straight line of stroke, the least possible in its intermediate positions. These deviations arise from the obliquity of the link to the line of stroke, therefore, each lever should be so placed, relatively to the link, that the greatest obliquity of the link shall be as small as possible, and for that purpose the link, in its positions of greatest obliquity to the right and left of the line of stroke, respectively, should make equal angles, with the line of stroke, on either side, which condition will be fulfilled by making the middle position of each lever, ct_2 and CT_2 perpendicular to the line of stroke, and also by making the line of stroke Ss bisect at M and m, the deflection T_2V and t_2v of the arcs described by the points T and t respectively, or as previously stated, the centres C and c, are so chosen, that the middle points of the versed sines of the same arcs are found on the line of stroke. This adjustment is to be made for the beam and link before designing the parallel motion, that is, we assume this principle of the versed sines.

206. Suppose in FIG. 11, PLATE 14, we have given the line of stroke GD and the length of stroke of a piston rod, the middle position of the guided point B, and the centre A of one lever, which in its middle position must be perpendicular to the line of stroke GD; to find the length of the lever AF so that the link connecting it with B shall deviate equally to the two sides of GD during the motion; also to determine the length of the link. It will be seen from the preceding figure that since $t_2m = mv$, from the assumed principle of the versed sines, the chord t_1t_2 of the upward arc of motion t_1t_2 will intercept on SS the line of stroke, a distance equal to $\frac{1}{2}t_1v = \frac{1}{4}$ the stroke. This follows from the consideration of the fact, that if we assume that the upward

swing of *Ct* shall equal its downward swing, and that $vm = t_2m$, it follows that $mx = \frac{1}{2}t_1v = \frac{1}{4}$ the stroke. From this consideration we make the following construction: in FIG. 11, PLATE 14, make $DE = \frac{1}{4}$ the stroke. Join *AE* and perpendicular to *AE* draw *EF*, cutting *AD* produced in *F*, *AF* will be the required length of the lever. Join *BF*, this will give the line of direction of the link in its central position. To determine the length of the link we must know the distance *GD* between the levers in their central positions; then through *G* draw *LK* parallel to *AF*, and prolong *FB* to *L;* then *FL* is the length of the link, and *LK* is the direction of the other lever. To find its length proceed exactly as in finding the length of *AF*. Lay off on *GD* a distance $GP = \frac{1}{4}$ the stroke; join *P* and *L* and draw *PK* perpendicular to *PL* and thus determine *K;* or make $DH = GB$. Join *AH* and prolong it until it cuts *LGK* in *K*, this will be the centre for the second lever. Again given by FIG. 12, PLATE 14, the middle positions, *AC* and *BE*, parallel to each other, and the extreme positions *AD*, *AD'*, and *BF* and *BF'*, of two levers centred at *A* and *B* respectively, required the length of these levers, so that a link connecting them shall deviate equally on opposite sides, of a perpendicular to *AC* and *BE*.

From *B* let fall *BG* perpendicular to *AC* produced. Draw *AH*, bisecting the angle *CAD;* the line of stroke is perpendicular to *AC* and *BE*, and cuts the line *AH* at such a point that a perpendicular to *AH* at that point, will intercept on *AC*, the required length of the lever centred at *A*. A line *BS*, bisecting the angle *EBF*, will determine similarly the length of the lever centred at *B*. The desired points are then those on the line of stroke, perpendicular to *AC* and *BE*, and likewise on the two lines *AH* and *BS*, bisecting the upward angle of motion of each lever.

Now make $PGH = EBS = \frac{1}{2}EBF$ and draw *GH* cutting *AH* at *H*. The line *PHN* perpendicular to *AC* and *BE* is the line of stroke and $4PH$ is the length of stroke. *HK* drawn perpendicular to *AH* and *HL* drawn perpendicular to *GH* determine respectively the points *K* and *L* as in case 1st, and hence the lengths of the levers *AK* and *GL;* make $BM = GL$, and draw *KM*. *KM*, is then the link connecting the two levers, and *R*, the intersection of *KM*, and the line of stroke, is the middle position of the guided point, but as the acuteness of the angle of intersection causes this method of finding R to be wanting in precision, it is better to proceed as follows: draw the line AB cutting NP in Q, then make $PR = NQ$ and R will be the required point.

207. The segments of the link, on either side of the point of attachment of the guided point, are, approximately, inversely proportional to the lengths of the nearest levers; this is the rule usually employed, when the extreme positions can vary a little from a straight line. When the levers are of equal lengths, this rule is exact. The greatest deviation of the guided point from the assumed line of stroke occurs when the link is parallel to the assumed line of stroke.

208. In the Watt's parallel motion let the perpendicular distance between the levers, in their middle position, be 18 inches; let the centres of the upper and lower levers be a perpendicular distance, of two and three feet, respectively, from the path of the guided point. Let this path be perpendicular to the two levers in their middle position. The stroke is 18 inches. To find the lengths of the two levers and the link and the position of the guided point.

In FIG. 9, PLATE 15, lay off $mn = 18$ inches, $Cn = 24$ inches and $Dm = 36$ inches and draw Cn and Dm perpendicular to mn. They then represent the direction of the levers in their middle position, and mn is the direction of the path of the guided point. The extent of this path is 18 inches; the middle and extreme positions of the guided point are on mn.

This travel of 18 inches is effected by the oscillation of the levers about their centres D and C, and each lever must swing an equal distance above and below Dm and Cn, their respective middle positions, in this case distances sufficient to produce a travel of 9 inches on each side of the middle position of the guided point, making the full stroke of 18 inches for the guided point, measured on mn. Then at mid stroke and extreme stroke, by the principle of the versed sines, the end of the link arm AB should be at equal distances from mn, and at ¼ stroke from mid position, the levers will cut the line of stroke at a distance from middle position = to ¼ stroke. In the figure make mS and $tn = \frac{1}{4}$ stroke $= 4.5$ inches, and connect S and t, thus determined, with D and C, respectively; draw perpendiculars to SD and Ct, and thus determine the points A and B, and thence, in length, the levers AD and BC and the link AB.

The point, where AB cuts mn, the line of stroke, is the position of the guided point P, at mid stroke.

Prolong AS and make $A''S = AS$, and also $B_1t = Bt$. Draw $B_1uB_{''}$ perpendicular to BC, making $B_{''}u = B_1u$; join $B_{''}$ thus determined with $A_{''}$, and the point P' where this line, $A_{''}B_{''}$ cuts the line of stroke mn, is the extreme upper position of the guided point.

Now to calculate lengths of the links and levers, we have by construction:

$$AD = Dm + \frac{AV}{2}$$

$$\text{hence } Dm = \frac{DA + DV}{2}.$$

also, since $A''v$ is a mean proportional between the segments into which it divides the diameter of the circle through A about D, we have

$$\overline{A''v}^2 = Av(DA + Dv) = Av \times 2Dm.$$

Calling the stroke, S, we have

$$Av = \frac{\overline{A''v}^2}{2Dm} = \frac{\left(\frac{S}{2}\right)^2}{2Dm} = \frac{S^2}{8Dm} \qquad (1)$$

By a similar process we have

$$Bu = \frac{S^2}{8Cn} \qquad (3)$$

Again $AD = Dm + Am$, and $Am = \dfrac{Av}{2}$

$$\therefore\ AD = Dm + \frac{S^2}{16\,Dm}, \text{ where } \begin{cases} S = 18 \text{ inches} \\ Dm = 36 \text{ inches.} \end{cases} \qquad (4)$$

$$\text{Similarly, } BC = Cn + \frac{S^2}{16\,Cn} \text{ and } \begin{cases} S = 18 \text{ inches} \\ Cn = 24 \text{ inches.} \end{cases} \qquad (5)$$

$$\text{From the triangle } AaB,\ \overline{AB}^2 = \overline{Aa}^2 + \overline{Ba}^2 \qquad (6)$$

$$Aa = mn, \text{ and } Ba = Bn + na;\ na = mA.$$

$$Bn = \frac{Bu}{2} \text{ and } mA = \frac{vA}{2}.$$

$$\therefore \text{ from (6) } \overline{AB}^2 = mn + \left(\frac{Av}{2} + \frac{Bu}{2}\right)^2 \qquad (7)$$

$$\therefore AB = \left(\overline{mn}^2 + \left(\frac{S^2}{16Dm} + \frac{S^2}{16Cn}\right)^2\right)^{\frac{1}{2}} \qquad (8)$$

By substituting for these known quantities their numerical values, we have

$AD = 36.562$ inches,
$BC = 24.843$ inches,
$AB = 18.03$ inches.

To find the position of the guided point, analytically, we have the following proportion :

$$AP : PB :: Am : Bn :: 2AM : 2Bn :: Av : Bu$$

Equations (1) and (3) give $Av = \frac{S^2}{8Dm}$, and $Bn = \frac{S^2}{8Cn}$

$$\therefore \quad AP : PB :: \frac{S^2}{8}, \frac{1}{Dm} : \frac{S^2}{8}, \frac{1}{Cn} :: Cn : Dm$$

From the above proportion we have,

$$AP : AP + PB :: Cn : Cn + Dm,$$

whence by substituting AB for $AP + PB$

$$\text{we have } AP = \frac{AB \times Cn}{Cn + Dm} = \frac{18.03 \times 24}{24 + 36}$$

$$\text{or } AP = 6.$$

which value of AP determines the position of P.

209. A piston rod moves vertically in a given direction, the pivot of the lower lever is 30 inches from the line of direction of the path, the levers in middle position are 20 inches apart, the stroke is 18 inches, the middle position of the guided point is on the line of direction of the path given, and 9 inches above the line of the middle position of the lower lever.

Construct a Watt's parallel motion, to guide the given point so that its extreme and middle positions shall be on the given line, and determine and calculate the proportions of the parts.

In FIG. 1, PLATE 15, we have the line *SS* in direction and the position of the point *D* given, *Dm* being 30 inches in length and perpendicular to *SS*. Lay off $mn = 20$ inches, and through *n* thus determined, draw a perpendicular to *SS*, it is the line of direction of the upper lever in its middle position; lay off $mP = 9$ inches and it is the middle position of the guided point. Determine *AD* as in the previous cases, and from *A* thus determined, through *P* draw the straight line *APB* and determine *B*, the intersection of *APB* with the perpendicular to *SS* through *n;* then *AB* is the link, and *B* is one end of the upper lever; the centre *C* of this lever can be found as in the previous case; or lay off $bn = \frac{1}{4}$ the stroke, join *B* and *b* and draw *bC* perpendicular to *Bb*, the point *C* where it cuts the perpendicular to *SS* through *n* is the desired centre, and the length of this lever is *BC*.

It remains then to calculate the lengths of *AD*, *AB*, and *AC*, which can be done from the given data, and by applying the formula already deduced for these values.

The parallel motion is then known in all its parts, and the necessary levers, links and pivots can be made of the correct dimensions to accomplish the required motion of the guided point, whose place of attachment to the link is known.

210. THE PANTOGRAPH. As the principle of the pantograph is made use of in the applications of Watt's parallel motion, it will be considered here.

The pantograph is a four-bar linkage *ABCD* (FIG. 2, PLATE 15), whose opposite links are equal, thus always forming a parallelogram. Two of the adjacent links as *AB* and *AD* are prolonged to convenient lengths.

Take a convenient point *S* on the link *AB* produced, and draw the straight line *SRP* intersecting the links *BC*, and *AD* prolonged, at *R* and *P* respectively. Now if we fix the point *S* and place pencils at the points *P* and *R* it will be found on moving the linkage that the pencils *P* and *R* will trace similar curves. For suppose *P* to move to *P'*, then the position of the linkage is shown by dotted lines, the line *ABS* being now found at *A'B'S*.

211. Now to prove that the curves *PP'* and *RR'* traced by the pencils are similar, it will only be necessary to show that *SP* and *SR* always have the same ratio to each other in all the positions

of the linkage. For the full position of the linkage we have by similar triangles $\frac{SP}{SR}=\frac{SA}{SB}$, and for the dotted position $\frac{SP'}{SR'}=\frac{SA'}{SB'}=\frac{SA}{SB}$; this ratio is constant for all positions of the linkage, and we will always have, when P and R are on the same straight line, which must be the case, similar curves traced by P and R; if PP' is straight, RR' will also be straight.

212. The pantograph is often used to enlarge or reduce drawings; in such a case if a reduction is desired, the drawing to be reduced is placed under the pencil P, and a piece of paper on which the reduced drawing is to be made is placed under R, both pieces of paper and the pivot S being fast to the board. On moving the pencil or tracer P over the lines in the drawing, the pencil R will produce a similar drawing on a smaller scale. The scales of the two drawings will be to each other as AS is to BS.

Pantographs are also often used to reduce or increase motions in a definite proportion, as in indicator rigs where the motion of the crosshead is reduced proportionally to the desired length of the indicator diagram.

213. When P and R are required to move in parallel straight lines it is not always necessary to employ a complete parallelogram, provided the mechanism is such that the points P and R are properly guided.

Such a case is shown in FIG. 3, PLATE 15, which shows the pencil movement of a Richards steam engine indicator. The pencil p which traces the diagram upon an oscillating drum is guided by the Watt parallel motion $dabc$, so that it moves in a line ss parallel to the axis of the indicator cylinder $s's'$. It must also be arranged that the pencil p has a motion in the line ss which always bears the same relation to the motion of the piston of the indicator. To have this the case draw the straight line $pp'c$ and note the point p' where this line cuts the centre line of the piston rod, which must always move in the line of the axis of the cylinder, as it is so guided by the cylinder cover and piston; then connect p' with the lever bc by means of a short link $p'r$, placed so that its centre line is parallel to that of ab. The motion of p is to the motion of p' as cb is to cr, which is a fixed ratio. Here it is not necessary to have the fourth link of the parallelogram, as the point p' is compelled to move in a straight line by other guides; the additional link would also be objectionable, as it would add weight to the reciprocating parts of the apparatus which should be made as light as possible.

214. APPLICATIONS OF WATT'S PARALLEL MOTION. Watt's parallel motion has been much used in beam engines, and it is generally necessary to arrange the motion so that more than one point can be guided, which is accomplished by pantograph attachment.

In FIG. 4, PLATE 15, a parallel motion is arranged to guide three points P, P', and P'' in parallel straight lines. The case chosen is that of a compound condensing beam engine, where $P''p''$ is the piston rod of the low-pressure cylinder, $P'p'$ that of the high-pressure cylinder, and Pp the pump rod, all of which should move in straight parallel lines, perpendicular to the centre of the beam CR in its middle position.

The fundamental linkage, $DA'BC$, is arranged to guide the point P as required; then by superadding the parallelograms $ASTB$ and $AP''RB$, and by placing the links TS and RP'' so that they pass through the points P' and P'' respectively, found by drawing the straight line CP and noting the points P' and P'' where it intersects $P'p'$ and $P''p''$, we obtain the required linkage. Here the links are arranged in two sets with the rods attached between them; the link DA' is also placed outside of the link $P''A$.

215. The different links are usually designated as follows: CR one-half of the *main beam*, $A'D$ the *radius bar or bridle*, $P''R$ the *main link*, $A'B$ the *back link*, $P''A$ the *parallel bar* connecting the main and back links.

In order to proportion the linkage so that the point P'' shall fall at the end of the link RP'' we have by similar triangles CBP and CRP'',

$$CB : BP = CR : RP'' = CR : AB.$$

$$\therefore CR = \frac{AB \times BC}{BP} \qquad (1)$$

216. To find the stroke of the point P'', that of P (s), being given, we have

$$S : s = CP'' : CP$$

$$S = s\,\frac{CP''}{CP} = s\,\frac{CR}{CB} = s\,\frac{M}{N}$$

$$\text{and, conversely,}\quad s = S\,\frac{N}{M} \qquad (2)$$

where M and N are the lengths of the perpendiculars Cx and Cz dropped from C upon the lines of motion of the points P'' and P respectively.

217. In the same way the stroke of P' may be expressed in terms of that of P'' or P.

The problem will generally be, given the centres of the main beam C and the bridle D, the stroke S of the point P'' and the paths of the guided points P, P', and P'', to find the remaining parts. The strokes of the guided points can be found from the equation (2), and then the general solution as explained can be applied.

218. FIG. 5, PLATE 15. To design a pantograph that would enable you to reduce drawings to ⅓ their size and explain its construction, draw the parallelogram $ABCD$ and make AD and BC = one inch. Prolong AD a convenient distance to T, make $AS = 3$ inches and $BS = 1$ inch, mark the point M where a line from T to S cuts BC. If then 4 links be made of some rigid material, of these dimensions and pivoted together at A, B, C, and D according to this construction, and plan, and a tracer be fastened at T and a pencil at the point M, as determined, this pencil at M will reproduce, reduced ⅓ in all its parts, any drawing traced over by T or even any motion imparted to T.

For in any and all positions

$$ST : SM :: AS : BS :: 3 : 1.$$

Take any other position, as the one designated, and we will still have

$$ST' : SM' :: SA' : SB' :: AS : BS :: 3 : 1$$

hence in all positions the points T and M will hold similar positions and M will reproduce the motion of T reduced ⅓.

219. FIG. 7, PLATE 15. Having given one-half the beam *abcd*, of a compound condensing beam engine, with two cylinders and pump attachment, where $P''p''$ is the piston rod of the low pressure cylinder, $P'p'$ that of the high pressure cylinder, and Pp the pump rod, all of which should move in straight parallel lines, perpendicular to the centre line of the beam in its middle position, CR the centre line, with the centre of motion of the beam at C, AD the radius bar with its centre at D, there being 18 inches between AD and CR in their middle positions and D and C being 60 inches apart, measured parallel to AD and CR in their middle positions, *xy*, *qz*, and *st* the paths of the guided points P'', P' and P, these paths being perpendicular to CR in its middle position, and 66, 54 and 36 inches, respectively, from the vertical through C, S, the stroke of P'', 24 inches, required the dimensions and arrangements of a Watt's parallel motion, with pantograph, to properly guide the given points.

In FIG. 7, PLATE 15, we have given:

S, the stroke of P'',	24 inches,
so,	18 inches,.
Cr,	60 inches,
Cx,	66 inches,
Cz,	54 inches,
Cs,	36 inches,
Do,	24 inches,
sz,	18 inches,
zx,	12 inches,

by assumption, *xy*, *qz*, and *st*, the directions of the paths of the guided points are perpendicular to CR, the centre line of the beam in its middle position, and AD in its middle position, is parallel to CR in its middle position, as shown in FIG. 7.

1st. To find the elements of the Watt's parallel motion $DABC$. The discussion of this mechanism gives the following expressions for the values of the required elements.

s = the stroke of P, to be determined.

$$AD = Do + \frac{s^2}{16Do} \tag{1}$$

$$BC = Cs + \frac{s^2}{16\,Cs} \qquad (2)$$

$$AB = \left(\overline{so}^2 + \left(\frac{s^2}{16\,Do} + \frac{s^2}{16\,Cs}\right)^2\right)^{\frac{1}{2}} \qquad (3)$$

$$PA = \frac{AB \times Cs}{Do + Cs} \qquad (4)$$

$$PB = BA - PA \qquad (5)$$

220. All the quantities in the second members of these five equations are given, except s, the stroke of the guided point P, which can be determined from the given stroke of P'', 24 inches, and the known distances of the paths of P'' and P, respectively, from C, the common centre of motion,

$$\text{for the stroke of } P'' : \text{stroke of } P :: Cx : Cs,$$

$$\text{or } 24 : s :: 66 : 36,$$

$$\therefore \text{ the stroke of } P \text{ (or } s) = 13.1 \text{ inches.}$$

By making the substitutions, in equations (1), (2), (3), (4) and (5), the lengths of the parts of the parallel motion become known; likewise the position of the guided point P and its proper point of attachment to the link AB.

To find the point of attachment of P'' we have

$$Ps = \sqrt{BP^2 - Bs^2},$$

in which BP is known and $Bs = BC - Cs$, both being known,

$$\text{again } P''x : Ps :: Cx : Cs$$

$$\text{and } P''x = \frac{Cx \times Ps}{Cs},$$

this gives the distance from x, on the given direction xy, to the point of attachment of P''; through P'' draw $P''A'$ parallel to CR and prolong AB to A', draw MP'' parallel and equal to $A'B$, join C and P''. Then,

$$CP'' = \frac{CP \times Cx}{Cs} \text{ and } CM = \frac{CP'' \times BC}{CP}$$

These expressions locate P''; P' can be similarly located, and all the parts can be determined in dimension and location.

221. To construct graphically; from o lay off a distance $ol = s/4 = 3.25$ inches, being ¼ of the stroke of P, connect D and l and draw Al perpendicular to lD and thus determine A, and the length of the lever AD.

Make the same construction for BC, by laying off $sk = $ ¼ the stroke of P and proceeding in a similar manner. Draw DA, AB, and BC, and this will give the Watt's parallel motion required. The intersection of the link AB (in its middle position) with the line of direction of the motion of the point P, will give P, the middle position of the guided point. Connect C and P by a straight line, and prolong it, cutting the lines of direction of the motions of the other guided points at P' and P'' Let us assume the attachment of P'' at the end of the pantograph attachment and at the end of the link MP''.

Then draw MP'' parallel to AB as determined, prolong AB, and through P'' draw $A'P''$ parallel to CR in its middle position; this is the superadded parallelogram, or pantograph attachment. Through P' as determined draw FH parallel to $A'B$, and draw the rods, vertically, from P, P' and P'', and the construction is complete. C and D are fixed. B, M, F, P, P', P'', and A, A', H are all pivoted connections of the rigid pieces that intersect each other at these points. The oscillation of the beam about C as a centre will now cause the piston and pump rods attached at P P' P'' to move in parallel paths proportional in extent to their distances from C, under the combined action of the Watt's parallel motion and the superadded pantograph.

The figure is not drawn to scale.

222. ROBERT'S APPROXIMATE PARALLEL MOTION. This might also be called the W parallel motion, and is shown in FIG. 8, PLATE 15. It consists of a rigid triangular frame ABP forming an isosceles triangle on AB, the points A and B being guided by links $AD = BC = BP$, oscillating on the centres D and C respectively, which are on the line of motion DC.

To lay out the motion, let DC be the straight line of the stroke along which the guided point P is to move approximately, and P the middle point of that line. Draw two isosceles triangles, *DAP*, *CBP*, with sides $DA = AP = BP = BC$; join *AB*, which must be equal to *DP* and to *PC*. Then *ABP* is the rigid triangular frame, *P* the guided point, *D* and *C* the centres of the two links. The extreme positions when *P* is at *D* and *C* are marked DA_1A_2 and CA_2B_2, the point A_2 being common to both positions.

The length of each side of the triangle, as *DA*, should not be less than $1.186DP$, as in this case the points $C\ A_2\ A_1$ lie in one straight line as do also the points $D\ A_2\ B_2$ It may be made as much greater as space will permit, and the greater it is made the more accurate will be the motion.

223. The proportions of the sides and bases of the triangles that form the Robert's parallel motion must be such that *AD* shall be greater than 1.186 times *AB*. For if *AD* equals 1.186 times *AB*, as will be proved, when the point *P* coincides with *D* in the motion of the combination, the lines *AB* and *BC* will form one and the same straight line, this action produces a form of toggle joint at *B* (Fig. 6) and produces too great a thrust on the pivots, and leaves the combination prone to remain at rest, as the lever *BC* and the link *AB* are in a line passing through a centre of motion *C*. In Fig. 6, the point *P* is supposed to coincide with *D*, and the lines *AB* and *BC* of Fig. 8 are supposed to form the straight line *ABC* (Fig. 6). Required the proportion that must exist between the lengths of *AB* and *BC*, in order to permit *AB* and *BC* to swing into a straight line, when *P* coincides with *D*.

In Fig. 6 $AD = BD = BC$

From Fig. 8 $DC = 2AB$.

Draw *Dx* (Fig. 6) perpendicular to *AB*, and as *ADB* is an isosceles triangle, *Dx* will bisect *AB*, and

$$Bx = \frac{AB}{2}$$

$$\text{Then } \overline{Dx}^2 = \overline{DB}^2 - \overline{Bx}^2 \qquad (1)$$

$$\text{and } \overline{Dx}^2 = \overline{DC}^2 - \overline{Cx}^2 = 4\,\overline{AB}^2 - \overline{Cx}^2 \qquad (2)$$

$$Cx = BC + Bx = DB + Bx.$$

$$\overline{Cx}^2=\overline{DB}^2+2DB\times Bx+\overline{Bx}^2=\overline{DB}^2+DB\times AB+\frac{\overline{AB}^2}{4}$$

equating (1) and (2) and substituting we have

$$4\overline{AB}^2-\overline{DB}^2-DB\times AB-\frac{\overline{AB}^2}{4}=\overline{DB}^2-\frac{\overline{AB}^2}{4} \qquad (3)$$

$$2\,\overline{AB}^2=\overline{BC}^2+\frac{BC\times AB}{2}\text{, since } DB=BC$$

$$\text{or } \frac{33}{16}\overline{AB}^2=\overline{BC}^2+\frac{BC\times AB}{2}+\frac{1}{16}\overline{AB}^2$$

$$\frac{AB}{4}\sqrt{33}=BC+\tfrac{1}{4}AB$$

$$BC=\frac{AB}{4}\left(\sqrt{33}-1\right)=AB\left(\frac{5{\cdot}744-1}{4}\right)=1.186\,AB$$

So that when the sides AB and BC form one and the same straight line, when P coincides with D, $BC=1.186\times AB$; any larger value of BC, as stated, will give better results. The least value possible of BC, is the one determined. In FIG. 8 $BC=1.2\times AB$.

The two extreme positions of P, D and C, and the middle position P are on the same straight line, but the intermediate positions vary somewhat from that line.

224. TCHEBICHEFF'S APPROXIMATE PARALLEL MOTION. FIG. 10, PLATE 15, shows another close approximation to a straight line motion invented by Professor Tchebicheff of St. Petersburg. It is an application of the double rocking lever, with the levers crossed, CD being the fixed link.

The links are in the following proportion: let $CD=4$,

$$\text{then } AC=BD=5, \quad \text{and } AB=2.$$

The guided point P is located midway between A and B on the link AB. It can easily be seen when the point P is on the perpendiculars to CD from D and C that it is on the line PP' parallel to CD, as

$$DP'=\sqrt{25-9}=4=DB'-B'P'.$$

The intermediate positions deviate slightly from the straight line. With the arrangement shown in the figure the motion $2PP'$ would not be possible if a rod were placed at P, but by properly shaping the levers AC and BD it might be obtained.

PLATE XVI.

Intermittent Linkwork.—Intermittent Motion.

225. A reciprocating motion in one piece may cause an intermittent circular or rectilinear motion in another piece. It may be arranged that one-half of the reciprocating movement is suppressed and that the other half always produces motion in the same direction, giving the ratchet wheel; or the reciprocating piece may act on opposite sides of a toothed wheel alternately, and allow the teeth to pass one at a time for each one-half reciprocation, giving the different forms of escapements as applied in timepieces.

226. RATCHET WHEEL. A wheel, provided with suitably shaped pins or teeth, receiving an intermittent circular motion from some vibrating or reciprocating piece, is called a *ratchet wheel.*

In FIG. 1, PLATE 16, *A* represents the ratchet wheel turning upon the shaft *a; C* is an oscillating lever carrying the *detent, click,* or *catch B,* which acts on the teeth of the wheel. The whole forms the three-bar linkage *acb.* When the arm *C* moves left-handed, the click *B* will push the wheel *A* before it through a space dependent upon the motion of *C.* When the arm moves back, the click will slide over the points of the teeth, and will be ready to push the wheel on its forward motion as before; in any case, the click is held against the wheel either by its weight or the action of a spring. In order that the arm *C* may produce motion in the wheel *A,* its oscillation must be at least sufficient to cause the wheel to advance one tooth.

227. It is often the case that the wheel *A* must be prevented from moving backward on the return of the click *B.* In such a case a fixed *pawl, click* or *detent, D,* turning on a fixed pin, is arranged to bear on the wheel, it being held in place by its weight or a spring. FIG. 2 shows a retaining pawl which would prevent rotation of the wheel *A* in either direction; such pawls are often used to retain pieces in definite adjusted positions.

228. If the diameter of the wheel *A* (FIG. 1) be increased indefinitely, it will become a rack which would then receive an intermittent translation on the vibration of the arm *C:* a retaining

pawl might be required in this case also to prevent a backward motion of the rack.

229. A click may be arranged to push as in Fig. 1, or to pull, or to pull and push as in Fig. 2. In order that a click or pawl may retain its hold on the tooth of a ratchet wheel, the common normal to the acting surfaces of the click and tooth, or pawl and tooth, must pass *inside* of the axis of a pushing click or pawl, and *outside* the axis of the pulling click or pawl; the normal might pass through the axis, but the pawl would be more securely held if the normal was located according to the above rule, which also secures the easy falling of the pawl over the points of the teeth; as regards the action between the teeth and the detent, we observe that the wheel must tend to hold the detent down by the pressure which it exerts, and that it will do so as long as the line of pressure on the surface *pr*, Fig. 3, falls below the centre *D*.

230. If the angle *qrp* (Fig. 3, Plate 16) were opened out much more, the perpendicular upon *pr* might rise above *D*, and the detent would then fail to hold the wheel. Further the click has to return by slipping over the points of the teeth. The condition for this result is that the normal *tt* to the surface *pp*, shall fall between *D* and the centre of the wheel. Where very little force is required to hold the wheel, and the exact position is of consequence, as in counting machinery, the teeth may be pins, and the detent or click may be a roller pressed against them by a spring as in Fig. 4, Plate 16, where *FFF* are the pins on the wheel *A*, *C* is the vibrating arm, *B* the detent, *p* is the roller, and *s* the spring that presses *p* against *FFF*.

231. It is sometimes necessary, or more convenient, to place the click-actuating lever on an axis different from that on the ratchet wheel; in such a case care must be taken that in all positions of the click the common normal occupies the proper position: it will generally be sufficient to consider only the extreme positions of the pawl in any case; since when the lever vibrates on the axis of the wheel, the common normal always makes the same angle with it in all positions, thus securing a good bearing of the pawl on the tooth, it is best to use this construction when practicable.

232. *The effective stroke* of a click or pawl is the space through which a ratchet wheel is driven for each stroke of the arm. The total stroke of the arm should exceed the effective stroke by an amount sufficient to allow the click to fall freely into place.

233. A common example of the application of the click and ratchet wheel may be seen in several forms of ratchet drills used to drill metals by hand. As examples of the retaining pawl and wheel we have capstans and windlasses, where it is applied to prevent the recoil of the drum or barrel, for which purpose it is also applied in clocks.

234. A common method of applying a ratchet wheel in ordinary mechanism is to be found in its use in a *lifting jack.* The object of the ratchet is to drive a screw which accomplishes the traversing of the frame work of the jack, the friction is so great on the screw-thread that no detaining pawl or detent is necessary as there is no tendency to a backward motion.

In Fig. 5, Plate 16, the lever handle *EH*, which actuates the driving pawl, has a centre of motion coincident with that of the ratchet wheel, and the pawl *D* is held against the teeth by a spring *S*. On moving the handle to the right the pawl slips over the teeth, and on moving it to the left the spring presses the pawl upon the teeth, and the ratchet is advanced an angular distance, determined by the motion of the handle *EH*, and the screw head *R* is turned a corresponding amount, and the frame work is moved. This is the same mechanical contrivance employed in the ordinary angular ratchet brace or drill for drilling by hand through a limited angular distance, as for instance, drilling or boring a hole in such close proximity to a wall, that the ordinary brace with a motion through 360° cannot be employed.

235. The pawl and ratchet is applied commonly to *capstans, windlasses,* or *hand cranes* for raising weights. Its function is both to accomplish the conversion of reciprocating motion into circular motion, so that the consequent rotation of a drum and the winding thereon of a rope or chain may accomplish the useful overcoming of some resistance, and to prevent the recoil of the drum or barrel while the reciprocation takes place, in preparation for a new impulse or stroke.

236. It was a very early improvement to provide two pawls of different lengths, termed by the sailors *pawl and half pawl,* and thereby hold up the barrel or drum at shorter intervals during the winding on of the rope. In point of fact a ratchet of 8 teeth thus became practically equivalent to one of 16 teeth, at least as far as checking the recoil of the drum was concerned, and the men were thereby better protected from any injury that might be caused by the sudden and unexpected recoil of their handspikes. This idea is illustrated in FIG. 6, PLATE 16, where the two pawls differ in length by the space of half a tooth. As the wheel advances by intervals of half a tooth each pawl falls alternately, and the same effect is produced as if the number of teeth were doubled and there was only one pawl.

237. In the same way three pawls might be used, each differing in length by one-third the space of a tooth, and so the subdivision might be extended. In practice it may be required to move a ratchet wheel through certain exact spaces differing by small intervals. Where such is the case, it is better not to attempt a too minute sub-division of the teeth, as they become iable to break and wear away, and the action is uncertain, but recourse should be had to a method of placing 3 or 4 clicks upon the driving arm. In such a case the pawls or clicks will be increased in number and will act as drivers, instead of detents, being arranged, as regards length, in the direct order of their magnitude.

They may be placed on separate driving arms, but there is no advantage in doing so, and it is usual to place them all upon one pin at the end of the driving bar. In this latter case the thickness of the follower or ratchet wheel must be at least equal to the length of the pin referred to, that is, the thickness of the ratchet wheel must be equal to the combined thicknesses of the pawls.

238. In FIG. 7, PLATE 16, 2 clicks *DP* and *DQ* are shown hung upon a pin *D* which is supposed to be at the end of the arm which drives the wheel. The clicks differ in length by half the space of a tooth, and they will engage the wheel alternately, and will move it in the same manner as if there were twice the number of teeth, driven by one click, and so for 3 or any number of pawls or clicks.

239. It will be found that one of the chief uses of the pawl and ratchet in mechanism is to provide feed motions in machinery for cutting or shaping metals: These feed motions are automatic movements by which, if the cutting tool be stationary, the metal to be worked is advanced a certain definite distance, preparatory to the next action of the tool; or, if the metal to be worked is stationary with reference to the cutting tool, the tool is moved a certain definite distance, in preparation for its next action on the metal. The general adopted plan is to draw off, as it were, some definite amount of motion at proper intervals during the operation, and so to impart an unchangeable movement of vibration, through a definite angle to a bar, which is the first element in the *feed mechanism.*

We give this bar a fixed amount of vibration, a given oscillation, and we require an adjustable attachment thereunto by means of which we can take a greater or less movement therefrom so as to advance the ratchet wheel a greater or less angular distance.

240. Suppose in FIG. 8, PLATE 16, that AC is a vibrating arm which swings with a definite amount of angular motion through the space PCQ. If AC be connected to another point S by the link RS, we can impart to the point S a reciprocating motion in the line CS, which may be measured by the length of the line EF. If now R, the point of attachment of the link RS to the vibrating bar AC, be moved towards C, the movement of the point S will be correspondingly diminished as R approaches C, becoming zero when R coincides with C. Similarly if R be made to recede from C, the travel of S will be correspondingly increased. Thus by means of a vibrating bar AC, oscillating through a known angle, and an adjustable link RS, attached at R to this bar, and at S to the click that gives movement to a ratchet wheel, we can at will impart given angular motions to this ratchet wheel, between the limits of zero and the extreme arc movement possible for the point R on the bar AC.

The amount of oscillation of the vibrating arm can in turn be regulated, as can be seen from the construction of a 4-bar linkage, wherein a rotation of one link causes an oscillation of another link, through angles that can be regulated, between the limits of 0 and 180° by varying the proportions of the parts. (See FIG. 4, PLATE 12.)

241. REVERSIBLE CLICK OR PAWL. The usual form of the teeth of a ratchet wheel is that given in FIG. 1, PLATE 16, which only admits of motion in one direction; but in "feed mechanisms", such as those in use in shapers and planers, it is often necessary to make use of a click or ratchet wheel that will drive in either direction. Such an arrangement is shown in FIG. 9, PLATE 16, where the wheel A has radial teeth, and the click, which is made symmetrical, can occupy either of the positions B or B', thus giving to A a right or a left-handed motion. In order that the click B may be held firmly against the ratchet wheel A, in all positions of the arm C, its axis, after passing through the arm, is provided with a small triangular piece f (shown dotted); this piece turning with B has a flat-ended presser always urged upward by a spring S, bearing against its lower face, opposite B, thus urging the click toward the wheel; a similar action takes place when the click is in the dotted position B'. When the click is placed vertically above the ratchet wheel A and in line with the arm C, it is held in position by the side of the triangle parallel to the face of the click; thus this simple contrivance serves to hold the click so as to drive in either direction, and also to retain it in position when thrown out of gear.

242. As for different classes of work a change in the "feed" is desired, we must arrange that the motion of the ratchet wheel A (FIG. 9, PLATE 16), which produces the feed, can be adjusted. This is often done by changing the swing of the arm C, which is usually actuated by a rod attached at its free end. The other end of the rod is attached to a vibrating lever which has a definite angular movement at the proper time for the feed to occur, and is provided with a T-slot in which the pivot for the rod can be adjusted by means of a thumb screw and nut. By varying the distance of the nut from the centre of motion of the lever, the swing of the arm C can be regulated; to reverse the feed, the click must be reversed and the nut moved to the other side of the centre of swing of the lever.

243. A feed motion extremely ingenious in principle has been applied in *Sir Joseph Whitworth's planing machines.* The principle is as follows:

In Fig. 10, Plate 16, let *CA* represent a vibrating bar, centred at *C* upon which point there is also centred a circle carrying two pins *P* and *Q*. We will suppose that the circle vibrates independently of the arm, through an angle exactly represented by *PCQ*, and that the object of the contrivance is to impart to the bar *CA*, the whole or any portion of this vibration. It is a simple matter to impart the entire vibration, for, if the pins *P* and *Q* are placed on either side of *CA* and close to it, they virtually make *CA* and the circle one piece, and consequently, they will vibrate together, or, any motion imparted to the circle will be imparted to the arm *CA*. Or if we wish the bar to remain at rest, we have only to separate the positions of the pins *P* and *Q* to such a distance that the angular distance between them equals or exceeds the angle *PCQ*, when the vibration of the circle through the given angle *PCQ*, will impart no vibration whatever to the arm *AC*. Conceive now that the pin *Q* is shifted to *q*, the arm *CA* will be pushed to *Q* by the pin *P* when moving to the right, but will only return as far as *Q* can carry it, that is to *q*, and the vibration of the arm *AC* will take place only through the angle *QCq*, while the circle vibrates through the angle *PCQ*, and in this way by adjusting the positions of the pins *Q* and *P*, the vibration of *AC* may be varied at will between the limits of *Q* and *QCP*. So we can obtain a feed motion that will advance the ratchet wheel through any desired angular distance, corresponding to any integral number of teeth, up to a limit fixed by the possible amount of the vibration of the arm. The practical arrangement is shown in Fig. 11, Plate 16, where a ratchet wheel, an arm carrying a click and another wheel provided with a circular slot, are placed in order named upon the same axis, and can always move independently of one another. There are two movable pins in the circular slot, which are capable of being fixed in any position by nuts at the back of the wheel. These pins project beyond the slot a sufficient distance to embrace the arm carrying the click only, but do not reach the click itself. The ratchet wheel is connected with a screw which advances the cutter across the table of the planer, and the object is to impart definite but varying amounts of rotation to the screw after each cut is taken. The table has a reciprocating motion; to the table is secured the metal to be worked; the mechanism that automatically reverses the motion of the table, imparts a given angular motion of vibration to the circle *E*. This wheel *E* then receives a fixed amount of vibration from the reversing of the table and will impart the whole thereof to the click *B*, if the adjustable pins *ee* are made to embrace the arm *C* closely on each side; or it will impart any less amount, gradually diminishing to zero, as the adjustable pins *ee* are separated to greater intervals in the slot that carries them, and thus the amount of feed of the cutter can be regulated. In FIG. 11 *E* is the driver, and the ratchet wheel (and consequently the shaft *a*) is the follower, *C* is an arm carrying the click *B*, and swinging loosely on the shaft *a*, which is fixed to the ratchet wheel *A*. The wheel *E*, also turning loosely on the shaft *a*, and placed just behind the arm *C*, has a definite angular motion sufficient to produce the coarsest feed desired; its concentric slot is provided with two adjustable pins *ee*, held in place by the nuts at their back ends, which can enclose the lever *C*, but are not of sufficient length to reach the click *B*. When the pins are placed at the ends of the slot, no motion will occur in the arm *C*; but when *e* and *e* are placed as near as possible to each other, confining the arm *C* between them, all of the motion of *E* will be given to the arm *C*, thus producing the greatest feed; any other positions of the pins will give motions between the above limits, and the adjustment may be made to suit each case.

244. FIG. 12, PLATE 16, shows another method of adjusting the motion of the ratchet wheel *A*. The stationary shaft *a*, made fast to the frame of the machine, carries the vibrating arm *C*, the ratchet wheel *A* and the adjustable shield *S*; the two former turn loosely on the shaft, while the latter is made fast to it by means of a nut, the hole in *S* being made smaller than that in *A*, to provide a shoulder against which *S* is held by the nut. The arm *C* carries a pawl *B* of a thickness equal to that of the wheel plus that of the shield *S*; the extreme positions of this pawl are shown by dotted lines at *B''* and *B''*. The teeth of the wheel *A* may be made of such shape as to gear with another wheel operating the

feed mechanism; or another wheel, gearing with the feed mechanism, might be made fast to the back of *A*, if more convenient: in the latter case, the arm *C* would be placed back of this second wheel.

If we suppose the lever in its extreme left position, the click will be at *B''* resting upon the face of the shield *S*, which projects beyond the points of the teeth of *A*; and in the right-handed motion of the lever the click will be carried by the shield *S* until it reaches the position where it will leave the shield and come in contact with the tooth *b*, which it will push to *B''*, right, in the remainder of the swing. In the backward swing of the lever the click will be drawn over the teeth of the wheel and face of the shield to the position *B''*, left. In the position of the shield shown in the figure a feed corresponding to two teeth of the wheel *A* is produced; by turning the shield to the left one, two, or three teeth, a feed of three, four, or five teeth might be obtained; while, by turning it to the right, the feed could be diminished, the shield *S* being usually made large enough to consume the entire swing of the arm *C*. This form of feed mechanism is often used in slotting machines, and in such cases, the click is usually held to its work by gravity.

245. DOUBLE-ACTING CLICK. This device consists of two clicks making alternate strokes, so as to produce a nearly continuous motion of the ratchet wheel which they drive, that motion being intermittent only at the instant of reversal of the movement of the clicks.

A form of a double-acting click has been devised in which two clicks are hung upon a vibrating bar, one of them pushes and the other pulls, to this end terminating in a hook. In FIG. 13, PLATE 16, the bar *ECD* vibrates on *C* as a centre, and the pieces *QD* and *PD* hang at the extremity *D*.

When *P* pushes on the ratchet wheel, the arms *PD* and *QD* open and the hook at *Q* slips over a tooth, whereas when the arm *ECD* oscillates, and moves the end *E* downwards, by the reversing of the motion of the driving lever *ECD*, the hook at *Q* pulls the wheel with it, *P* slips over a tooth, and thus the wheel advances one tooth for each vibration of the moving arm *ECD*, or for each complete oscillation of the driver, or for one revolution of the crank that imparts this oscillation, the ratchet wheel advances 2 teeth.

246. FRICTION CATCH. Various forms of catches depending upon friction are often used in place of clicks; these catches usually act upon the face of the wheel, or in a suitably formed groove cut in the face. Friction catches have the advantage of being noiseless, and allowing any motion of the wheel, as they can take hold at any point; they have the disadvantage, however, of slipping when worn, and of getting out of order.

247. FIG. 14, PLATE 16, shows a friction catch *B* working in a V-shaped groove in the wheel *A*, as shown in section at *A'B'*. Here *B* acts as a retaining click, and prevents any right-handed motion of *A*; its face is circular in outline, the centre being located at *d*, a little above the axis *c*. A similarly shaped catch might be used in place of an actuating click to cause motion of *A*.

248. FIG. 15, PLATE 16, shows four catches like *B* (FIG. 14) applied to drive an annular ring *A* in the direction indicated by the arrow. When the piece *c* is turned right-handed, the catches *B* are thrown against the inside *b* of the annular ring by means of the four springs shown; on stopping the motion of *c*, the pieces *B* are pushed, by the action of *b*, toward the springs which slightly press them against the ring and hold them in readiness to again grip when *c* moves right-handed. Thus an oscillation of the piece *c* might cause continuous rotation of the wheel *A*, provided a fly-wheel were applied to *A* to keep it going while *c* was being moved back. The annular ring *A* is fast to a disc carried by the shaft *a*, the piece *c* turning loosely on *a* has a collar to keep it in position lengthwise of the shaft.

249. THE NIPPING LEVER shown in FIG. 16, PLATE 16, is another application of the friction catch. A loose ring surrounds the wheel *A*; a friction catch having a hollow face works in a pocket in the ring and is pivoted at *c*. On applying a force at the end of the catch *B* in the direction of the arrow, the hollow face of the catch will "nip" the wheel, and cause the ring to bear tightly against the left-hand part of the circumference of the wheel; the friction thus set up will cause the catch, ring, and wheel to move together as one piece. The greater the pull applied at the end of the catch the greater will be the friction, as the friction is

proportional to the pressure; thus the amount of friction developed will depend upon the resistance to motion of *A*. Upon reversing the force at the end of the catch, the hollow face of the catch will be drawn away from the face of *A* and the rounding top part of the catch, coming in contact with the top of the cavity in the ring, will cause the ring to slide back upon the disc. An upward motion of the click end will again cause the wheel *A* to move forward, and thus the action is the same as in a ratchet and wheel.

250. Another form of friction catch, sometimes used in gang saws to secure the advance of the timber for each stroke of the saw, and called *the silent feed*, is shown in FIG. 17, PLATE 16.

The saddle block *B*, which rests upon the outer rim of the annular wheel *A*, carries the lever *C* turning upon the pin *c*. The block *D*, which fits the inner rim of the wheel, is carried by the lever *C*, and is securely held to its lower end by the pin *d* on which *D* can freely turn. When the pieces occupy the positions shown in the figure, a small space exists between the piece *D* and the inside of the rim *A*.

The upper end of the lever *C* has a reciprocating motion imparted to it by means of the rod *E*. The oscillation of the lever about the pin *c* is limited by the stops *e* and *G* carried by the saddle block *B*. When the rod *E* is moved in the direction indicated by the arrow, the lever turning on *c* will cause the block *D* to approach *B*, and thus nip the rim at *a* and *b;* and any further motion of *C* will be given to the wheel *A*. On moving *E* in the opposite direction the grip will first be loosened, and the lever striking against the stop *e* will cause the combination to slide freely back on the rim *A*. The amount of movement given to the wheel can be regulated by changing the stroke of the rod *E* by an arrangement similar to that described in connection with the reversible click. The stop *G* can be adjusted by means of the screw *F* so as to prevent the oscillation of the lever upon its centre *c*, thus throwing the grip out of action. The saddle block *B* then merely slides back and forth on the rim, the action being the same as that obtained by throwing the ordinary click out of gear.

PLATES VIII AND IX.

Toothed Wheels.

251. Since rolling cylinders or cones, connected by frictional contact, cannot be relied upon to transmit much force, and at the same time maintain a constant velocity ratio, they are provided with interlocking teeth, thus forming *gear wheels.*

252. Pitch surface. The pitch surface of a toothed wheel or rack is an ideal smooth surface situated between the tops and bottoms of the teeth, which, by rolling contact with the pitch surface of another wheel, would communicate the same velocity ratio as the teeth communicate by sliding contact. The pitch surfaces may be generated by a straight line or element considered as moving along a line or curve as a *directrix.* This directrix is called the *pitch line.*

253. Pitch line. The pitch line may be otherwise defined, in most cases which occur in practice, as the trace of the pitch surface on a plane perpendicular to the axis of rotation. In bevel gearing the largest intersection of the perpendicular plane, or the base of the rolling cone, is commonly called the *pitch circle.*

254. Pitch point. The pitch point of a pair of gear wheels is the point of contact of their pitch lines, or the trace of the line of contact of the surface containing the pitch lines. In Fig. 1, $AA_{\prime}$ and $BB_{\prime}$ are the pitch circles, and C is the pitch point.

The pitch line is divided into as many equal parts as there are to be teeth upon the wheel, each part containing a tooth and a space.

255. Pitch. The pitch of a gear wheel is the distance *measured on the pitch line* from a point on one tooth to the corresponding point on the next tooth, as CE (Fig. 1). The pitch is divided into two parts, CF, the tooth, and FE, the space. In all cases the pitch should be an aliquot part of the pitch line, and in order that wheels may gear with each other they must have the same pitch.

256. Backlash. To allow for inaccuracy of construction and to avoid the jamming of teeth, the space EF is made greater than

the width of the tooth CF; the difference, or $EF - CF$, is called the *backlash* or *play.* The amount allowed varies in different cases and depends on the material used, the method of forming the teeth, and the judgment of the designer. In cut gears the spaces are often made equal to the teeth, or very nearly so, thus giving practically no backlash. When this is the case, the teeth must be very accurately spaced and cut, or jamming will result.

257. ADDENDUM CIRCLE. A line, DD (FIG. 1), drawn through the tops of the teeth parallel to the pitch line is called the *addendum line*, or, in a circular wheel, the *addendum circle.*

258. ROOT CIRCLE. A line, RR (FIG. 1), drawn through the bottoms of the spaces parallel to the pitch line is called the *root line*, or, in a circular wheel, the *root circle.*

259. LENGTH. The length of a tooth is the distance between the addendum and root circles measured on a radial line, as l, FIG. 1.

260. BREADTH. The breadth of a tooth is the distance, measured along an element of the pitch surface, between the two bounding surfaces, of the tooth.

261. CLEARANCE. The distance $d = d_{\prime}$ (FIG. 1), between the pitch and root circles, is made a little greater than $h = h_{\prime}$ between the pitch and addendum circles, to prevent the end of one tooth from striking the bottom of the corresponding space. This difference $d - h_{\prime} = ab$, is called *the clearance.* One wheel may have only projections $h_{\prime}$ beyond the pitch line, while the other has corresponding spaces s inside its pitch line, the clearance being, as before, $d - h_{\prime}$; but this same clearance must also be allowed inside the pitch circle of the first wheel.

Parts of the Teeth.

262. FACE AND FLANK. That part of the front or acting surface of a tooth which projects beyond the pitch surface is called a *face*, as Cf and Cg (FIG. 1); that part which lies within the pitch surface, the *flank*, as Ca and $Ca_{\prime}$. That part of the flank which comes in contact with the corresponding face will be called *the acting flank.* The flanks of the teeth of the driver O drive the faces of the teeth of the follower during the approaching action of the teeth, and the faces of the teeth of the driver drive the

flanks of the teeth of the follower during the receding action of the teeth.

The *pitch point of a tooth* is the point where the pitch line cuts the front, or acting side, of a tooth, as E and C, FIG. 1.

Contact.

263. THE POINT OF CONTACT is the point where the teeth touch each other.

THE PATH OF CONTACT is the curve described by the point of contact during the entire action of a pair of teeth, which curve always passes through the pitch point.

264. ARC AND ANGLE OF ACTION. The angle through which a wheel turns, from the time when one of its teeth comes in contact with the engaging tooth of the other wheel until the point of contact has reached the line of centres, is called the *angle of approach;* the angle through which it turns from the instant that the point of contact leaves the line of centres until the teeth quit contact, is called the *angle of recess.* The sum of these two angles is the *angle of action.* The arcs of the pitch circle which measure these angles are called the *arcs of approach, recess,* and *action,* respectively. The corresponding arcs must evidently be the same for both pitch circles, but the corresponding angles are inversely proportional to the diameters of the pitch circles.

In order that one pair of teeth shall be in contact until the next pair begin to act, the arc of action must be at least equal to the pitch, and in practice it ought to be considerably greater.

It has been found in practice that the friction of approaching action is greater and more injurious than that in receding action.

265. THE LINE OF ACTION OR LINE OF CONNECTION between two gear teeth is the common normal to both tooth surfaces at the point of contact.

THE OBLIQUITY OF ACTION is the angle made by the line of connection with the perpendicular to the line of centres at the pitch point.

Gearing Classified.

266. There are six varieties of toothed gearing to be met with in practice, viz:

1. Spur Gearing,	4. Twisted Gearing,
2. Bevel Gearing,	5. Screw Gearing,
3. Skew Gearing,	6. Face Gearing.

1°. SPUR GEARING (FIG. 2). Here the axes of the wheels are parallel to each other. If we suppose the number of teeth to be increased indefinitely, their size being at the same time correspondingly diminished, they will finally become mere lines, or elements of surfaces in contact, thus giving the figure at the right; which figure shows the pitch surfaces, or the surfaces which would give the same velocity ratio by rolling contact as the corresponding gears give by sliding contact. The pitch surfaces are shown at the right in each of the following cases:

If the teeth of one wheel are replaced by pins, and the teeth of the other wheel are made to work properly with such pins, we have what is called *pin gearing.*

Should the radius of one of the gears be made infinite, the pitch line becomes a straight line, and we have a *rack.* The above cases may be considered as coming under the head of spur gearing.

2°. BEVEL GEARING (FIG. 3). Here the axes intersect, and the pitch surfaces are cones, having their common vertex at the point of intersection. When the axes intersect at right angles and the wheels are equal, they are termed *mitre wheels.*

3°. SKEW GEARING (FIG. 4). Here the axes lie in different planes, and the pitch surfaces are hyperboloids of revolution. This class of gearing is not generally used, owing to the difficulty of forming the teeth, it being possible in most cases, to make the connection by means of two sets of bevel gears. They are hard to cut and give much sliding between gears.

In cases 1, 2 and 3, the teeth touch each other along straight lines, parallel to the axis in 1, passing through the apices of the pitch cones in 2, and approximating in their general direction to the common element of the two hyperboloids (FIG. 4).

4°. TWISTED GEARING (FIG. 5). If we suppose one of a pair of engaging circular wheels, belonging to either of the first three classes, to be uniformly twisted on its axis, each successive transverse plane being rotated through a greater angle, the other wheel receiving a corresponding twist as shown, we will have a case of twisted gearing. The action is gradual and continuous and jars are avoided.

The wheels thus formed will gear together as well as before. The teeth have faces of a helicoidal nature, and by increasing the number of teeth indefinitely, helical lines or elements would result, giving the same pitch surface as before.

In the case shown, pressure will result along the axes of the gears. To neutralize this axial pressure, the twisting can be made to start at the central plane of the wheel, and proceed the same on each side, as shown at *A*.

5°. SCREW GEARING (FIG. 6). Here the teeth also have a helicoidal form, as in twisted gearing, and reduce to helical lines; but these helices lie upon cylinders whose axes are in different planes, the pitch surfaces touching in a single point only. As illustrated by the "worm and wheel", it is the screw-like action alone of one wheel on the other which transmits the motion.

6°. FACE GEARING (FIG. 7) is not much used in modern machinery. The teeth generally consist of turned pins projecting from circular discs, but may be arranged on other surfaces than planes: the axes also may be inclined to each other.

In this case the teeth are circular in section, and touch only in one point; and when the number is increased indefinitely, two circles touching each other at their circumference will result.

Face wheels, then, have no pitch surfaces properly so-called, but surfaces of some kind are required to support the teeth. This class of gearing was best adapted to woolen mill machinery, and at one time was used almost exclusively for that purpose.

Angular Velocity Ratio of a Sliding Pair.

267. In FIG. 8, let $O_{\prime}$ and O be the fixed centres of motion of two pieces in sliding contact at the point C, their common tangent being Cc, and their common normal nn. The motion CA

of the point C, of the driver $O_{,}C$, may be resolved into two components, one along the common normal Ce, and one along the common tangent Cc. Now the driver $O_{,}C$ can act on the follower OC only along the common normal nn (the line of connection), and push it along that line as fast as it goes itself, and no faster; the component Cd along the common tangent and the motion CB, can be found by completing the parallelogram $CdBe$, CB being perpendicular to CO.

The common normal, or line of connection, cuts the line of centres $O_{,}O$ into two segments at the point h.

Let fall the perpendiculars $O_{,}g$ and Of on the common normal from the centres $O_{,}$ and O respectively, and let the angular velocity of the driver $O_{,}C$ be represented by V, and that of the follower by v. The linear velocity of any point of the driver $O_{,}C$ along the line nn is $V \times O_{,}g$; and the linear velocity of any point of the follower along the same line is $v \times Of$.

Since the motion of a point as C is the same for each piece, we have

$$V \times O_{,}g = v \times Of, \text{ or } \frac{V}{v} = \frac{Of}{O_{,}g}$$

From similar triangles $O_{,}gh$ and Ofh, we have,

$$\frac{OF}{O_{,}g} = \frac{Oh}{O_{,}h} \therefore \frac{V}{v} = \frac{Oh}{O_{,}h}$$

The angular velocity ratio is thus the same as that for rolling cylinders; hence the pieces in sliding contact at the point C give, for the instant, the same angular velocity ratio as a pair of rolling cylinders in contact at the point of intersection h of the line of connection and the line of centres.

§ 268. Another proof is as follows: By construction the triangles CAe and $O_{,}C_{,}g$ are similar, as are also CBe and OCf. Then we have

$$\left.\begin{aligned} V &= \frac{CA}{O_{,}C} = \frac{Ce}{O_{,}g} \\ v &= \frac{CB}{OC} = \frac{Ce}{Of} \end{aligned}\right\} \therefore \frac{V}{v} = \frac{Of}{O_{,}g} = \frac{Oh}{O_{,}h} \text{ as before}$$

269. Constant velocity ratio. From the above we conclude that the velocity ratio will be constant so long as the point h remains fixed. Hence the *necessary* and *sufficient* condition in order that a pair of gear teeth, acting by sliding contact, may give the same angular velocity ratio as the pitch circles, acting by rolling contact, is that the tooth curves at every point of contact shall have their common normal passing through the pitch point.

In Fig. 9, lettered the same as Fig. 8, the lower piece is the driver, and the point h falls on the same side of both centres instead of between them, as in Fig. 8, giving a case of internal gearing.

In Fig. 9 let r and s be the points consecutive to and on opposite sides of C. Then O_1C is greater than O_1r but less than O_1s; and thus we conclude that to produce compulsory rotation, the curves and their motions must be such that the contact radius of the driver shall be on the increase.

270. Rate of sliding. In Figs. 8 and 9 the components Cc and Cd represent the linear velocity of the points of contact along the common tangent. When these components lie in the same direction, as in the figures, their difference cd represents the rate of sliding of the two curves upon each other at the instant in question; when they lie in opposite directions, their sum represents the rate of sliding.

271. Pure rolling contact. In order that there may be no sliding between the surfaces, the tangential components must be equal and must have the same direction, the normal components being equal also.

To have this the case, the motion lines CA and CB (Fig. 8) must coincide; since the contact radii O_1C and CO must be perpendicular to this common motion line, they will coincide and form the line of centres. That is, for rolling contact, the point of contact must always lie upon the line of centres.

272. It will be noticed that the common normal nn (Fig. 8) corresponds to the connecting rod in a four-bar-linkage, the motion obtained from the linkage O_1gfO by means of the links O_1g and Of, being the same as that obtained by the sliding pieces for the instant.

273. CONJUGATE CURVES. Any two curves so related that, by their sliding contact, motion will be transmitted with a constant velocity ratio, as in rolling cylinders, are called conjugate curves. The curves in this case are often called *odontoids*.

Clearing Curve.

274. The "acting flank" of a tooth is usually shorter than the face of the tooth with which it gears; consequently the space between the two teeth must be cut beyond the flank deep enough to clear the point of the interlocking tooth. Let FIG. 10 represent a pair of teeth whose pitch circles are $AA_\prime$ and $BB_\prime$ and whose pitch point is C. Also let the path of contact be DCE. Consider the flank GG and the face CT. To find the point G, which limits the acting flank, draw the addendum circle TE intersecting the path of contact at E, and through E draw the arc EG from the centre of the pitch circle $AA_\prime$ intersecting the flank at G. The tip T of the interlocking tooth will, during the engagement of the face and flank, describe a curve called *the epitrochoid* (see definition, Art. 281) on the plane of the circle $AA_\prime$. To construct the epitrochoid, make the arc $Cb = Ca$, and with the radius $bt = aT$ strike an arc at t. In like manner strike the arcs t_1, t_2, and t_3, etc., from the points b_1, b_2, b_3, etc. The path of the point T will be a smooth curve touching these arcs on the inside, a sufficient number of points being taken on the pitch circle B to clearly fix the curve. This curve is the true clearing curve.

The clearing curve used in practice is an arc of a circle connecting the acting flank produced with the root circle and lying outside of the true clearing curve.

Drawing of Rolled Curves.

275. Any curve described by a point carried by one line which rolls upon another may be called a *rolled curve*.

The line which carries the tracing point is called the *generatrix*, *describing line*, or *describing circle*, while the one in contact with which it rolls is called the *directrix* or *base line;* either line may be straight, or both may be curved.

276. THE CYCLOID. This curve is traced by a point in the circumference of a circle which rolls upon a straight line as a directrix, as the curve $CC_4C_7C_{10}$ (FIG. 11) traced by the point C in

the circumference of the circle *Cc* which rolls on the straight tangent line $Cc_{\prime}$.

To construct the curve lay off on the circumference of the rolling circle a series of equal divisions as shown, and on the tangent line a similar series of divisions such that the divisions on the arc are of the same length as those on the straight line, and that the corresponding points will roll into each other. This division may be made by carefully using spacing dividers, setting them so fine that the difference between a chord and its arc is inappreciable, or by rectifying by calculation a portion of the arc of the rolling circle and dividing both into the same number of equal parts, as has been done in the figure, one-half of the circle being used. The centre of the rolling circle moves on a line $OO_{\prime}$, parallel to the tangent line, and its position can be found for each division by erecting perpendiculars through the divisions on the directrix. To find a point as C_7 corresponding to the seventh division ; here the centre of the rolling circle is at O_7, and the circle is tangent to the directrix at $c_{\prime\prime}$ (perpendicularly under O_7), the point *c* on the circle having rolled to $c_{\prime\prime}$ on the directrix. Hence striking an arc from O_7 with a radius *OC*, equal that of the describing circle, and intersecting this with an arc struck from the tangent point $c_{\prime\prime}$ with a radius equal to the chord *Cc*, we obtain the point C_7. In the same way other points may be found. The instantaneous centre of the rolling circle is at the tangent point $c_{\prime\prime}$, and $C_7c_{\prime\prime}$ is the normal, its length representing the radius of curvative of the cycloid at the point C_7. In fact, the cycloid might be drawn by making it tangent to a series of arcs struck as above from the divisions of the directrix.

Another method of locating the point C_7 is to draw through the point *c* on the circle a line cC_7 parallel to the directrix and noting where it intersects the corresponding arc of the rolling circle.

277. THE EPICYCLOID (FIG. 12). In this case the curve is traced by a point in the circumference of a circle which rolls on the *outside* of another circle as a directrix. Here the centre of the rolling circle moves on a circular arc parallel to the directrix, and the successive positions are found by drawing radii through the divisions of the directrix and noting the points o_1, o_2, o_3, etc.,

where they intersect the curve of centres. The construction is similar to that of the cycloid, and the figure is correspondingly lettered.

When the describing circle rolls on the outside of the directing circle, as in FIG. 12, we have an *external epicycloid;* when, however, the describing circle is greater in diameter than the directing circle and rolls internally upon it, as in FIG. 13, we have an *internal epicycloid.*

278. DOUBLE GENERATION OF THE EPICYCLOID. It will be found that the same epicycloid *C* (FIG. 14) will be generated by a circle *A* rolling on the directing circle *B* as will be generated by a circle *D*, whose diameter is equal to the sum of the diameters of *A* and *B*, rolling upon the same directing circle. This property of double generation of the epicycloid is made use of in discussing the tooth curves for internal gearing.

279. THE HYPOCYCLOID is traced, as shown in FIG. 15, by a point in the circumference of a circle rolling *inside* of another. The construction is the same as in the case of the epicycloid, and the figures are lettered the same throughout.

When the diameter of the rolling circle is one-half that of the circle in which it rolls, the hypocycloid becomes a diameter of the directing circle.

280. DOUBLE GENERATION OF THE HYPOCYCLOID. The hypocycloid may also be generated in two ways; thus in FIG. 16 the same hypocycloid *C* may be generated by the circle *A* rolling inside of *D* as is generated by the circle *B*, whose diameter is equal to the difference of the diameters of *D* and *A*, rolling inside of *D*. It will be noticed, however, that to trace the curve from the same starting point, the circles must roll in opposite directions from that point.

281. EPITROCHOID. The term epitrochoid is used in a general way to name curves traced by rolling one circle upon another when the marking point is not situated upon the circumference of the rolling circle. When the marking point is situated outside of the circumference of the rolling circle, the looped curve traced is known as a *curtate* epitrochoid; when it is situated inside, the curve is known as a *prolate* epitrochoid.

The above method of drawing the cycloidal curves may here be used; but the tracing point, not now being on the circumference of the rolling circle, must be located in its different positions by a method of triangulation from points whose positions are known.

282. THE INVOLUTE OF THE CIRCLE. This curve may be considered as generated by a point in a straight line which rolls upon a circle. It may also be regarded as generated by unwinding an inextensible fine thread from a cylinder; the thread being always taut and always tangent to the cylinder, its length is thus equal to the arc from which it was unwound.

To find one point as C_4 in the involute (FIG. 17): Draw the radius Oc, and perpendicular to it draw the tangent cC_4. Make the tangent cC_4 equal in length to the arc Cc, and C_4 is a point on the involute. The radius of curvature of the involute at C_4 is C_4c. It is generally most convenient to lay off equal divisions on the directing circle, and on some straight line a similar series of divisions equal to those of the circle. Then draw tangents at the divisions of the directrix, and on these lay off distances corresponding to the different arcs, the distances being taken from the divided straight line. From the construction it can be seen that the normal at any part of the curve is always tangent to the *base circle* or directrix.

Spur Gearing.—Epicycloidal System.

283. GENERATION OF THE TOOTH OUTLINE. In FIG. 18 let O and O_1 represent the centres of two pitch circles, AA_1 and BB_1, respectively, in contact at the pitch point C; and let a smaller circle, whose centre is o, be placed tangent to both pitch circles at C. Suppose now that the centres O, O_1, and o are fixed, then allow the circles to move in rolling contact with each other, the circle o carrying a marking point E_1. The point E_1 will, for the motion in the direction of the arrows, describe a hypocycloid d_1E_1 on the plane of the circle O extended. The arcs Cd_1 and Ce_1 are each equal to the arc CE_1, and E_1 is the tangent point of the two rolled curves, their common normal being E_1C. For any position as E we have the curves dE and eE, in contact at E, and the common normal EC.

284. Since the relative motions of the circles are the same as though the small one rolled on the outside of $BB_{\prime}$, and on the inside of $AA_{\prime}$ it is seen that the curves are the epicycloid of $BB_{\prime}$ and hypocycloid of $AA_{\prime}$; and since the common normal $E_{\prime}C$ always passes through the pitch point C for all positions of the sliding pieces from C to $e_{\prime}$, the curves would give the same angular velocity ratio by sliding as the pitch circles give by rolling during the interval $Ce_{\prime}$. The curves are then suited for gear teeth, $d_{\prime}k_{\prime}$ being a flank for the wheel O, and $e_{\prime}E_{\prime}$ a face for the wheel $O_{\prime}$. To get faces for O and flanks for $O_{\prime}$, a similar method could be used; that is, placing another circle tangent at C, with its centre between C and $O_{\prime}$, and proceeding as before.

285. To show that these two curves simultaneously generated will answer our purpose, let us conceive in FIG. 18*a* that we have described an epicycloid *aa* upon the outside of the pitch circle $AA_{\prime}$ and a hypocycloid *bb* upon the inside of the pitch circle $BB_{\prime}$, using the same generating circle in both cases, and let $AA_{\prime}$ and $BB_{\prime}$ represent the pitch circles of two discs on which teeth are to be cut. Let the circles be so placed that they touch as indicated, on the given line of centres, and let the two curves touch at some point as P. As this point was simultaneously generated, the two curves must have the same curvature at this point, and their common normal will pass through the point where the describing circle touches each of the pitch circles; for, the instantaneous axis is at D, and the point P, common to both curves, tends at the instant to turn about D; hence its path will be at right angles to PD for the instant, and PD will be normal to each curve at the instant and will be the radius of curvature of each curve at P; as P is any point common to the two curves, this principle applies wherever they touch each other.

286. It is to be observed that the conditions for a uniform velocity ratio would have been satisfied had the curves been traced by a marking point not in the circumference of the describing circle O; and it is to be observed that the tracing point need not be carried by a circle; any other describing curve that would roll on the pitch circles would fulfil the above conditions just as well, the resulting curves having a common point of tangency and a common normal at that point, which passes through the

pitch point. Hence we may in general say that any proper tooth outlines must be such as can be simultaneously traced upon the planes of rotation of the two wheels, while in action, by a marking point which is carried by a describing curve moving in rolling contact with both pitch circles. And conversely for any set of proper tooth curves there is a corresponding describing curve. ✓

Of the infinite number of tooth outlines that can geometrically satisfy the imposed conditions many are not practical; of the practical forms, none have been more extensively employed than the epicycloid and the involute, to which only attention is given.

287. *If in Epicycloid Spur Gears*, we designate one wheel as the driver and the other as the follower, the flanks of the driver and the faces of the follower must be generated by the same describing circle rolling inside of the driver's pitch circle and outside of the follower's pitch circle; and the faces of the driver and the flanks of the follower must be generated by the same describing circle rolling outside of the driver's pitch circle and inside of the follower's pitch circle. The two describing circles may or may not have the same diameter.

288. PATH OF CONTACT. From FIG. 18 it may be seen that the path of contact in epicycloidal gears is on the arc $CEE_{,}$ of the describing circle. Thus if in FIG. 19 we call $O_{,}$ the driver and O the follower, the path of contact begins at $D_{,}$, where the addendum circle of the follower meets the describing circle $o_{,}$, and ends at E, where the addendum circle of the driver meets the describing circle o. The flanks of the driver push the faces of the follower in the approaching action (path, arc $D_{,}C$); and the faces of the driver push the flanks of the follower during the receding action (path, arc CE). The arc of approach in this case is $e_{,}C = d_{,}C$, and that of recess is $Ce = Cd$.

From the above it can be seen that increasing the length of the teeth on either wheel increases the arc of action; and in any case, when the arcs of approach and recess are given, we can determine the necessary lengths of the teeth by laying off the arcs of approach, as $CD_{,}$, on the describing circle $o_{,}$, giving the point $D_{,}$, and the arc of recess on o giving the point E; through these points the addendum circles may then be drawn.

289. LIMITING CONDITIONS. Given the proportions shown in FIG. 19, to determine whether they are possible or not. Let it be assumed that there is no backlash, and that the teeth and spaces may be unequal.

The path of contact must in any case be at least equal to the pitch; the approaching action must be taken care of by the face of the follower, and the receding action by the face of the driver. If action begins at $D_{\prime}$, the flank of the driver $O_{\prime}$ will act on the face $D_{\prime}d_{\prime}$ of the follower until $d_{\prime}$ reaches C, the arc $d_{\prime}C$ being equal to $D_{\prime}C$. Draw the radial line $D_{\prime}O$, and note the point i where it intersects the pitch circle $AA_{\prime}$. If, now $d_{\prime}i$ were just one-half the thickness of the tooth of the follower, the tooth would be pointed, and would just secure the arc of approach $d_{\prime}C$; if it is greater than one-half the thickness, the case is impossible; and if less, as is possible in the figure, the tooth need not be pointed. Recess begins at C, and ends at E; the face of the driver acting on the flank of the follower, and the arc of recess $Cd = Ce = CE$. As before, draw the radial line $EO_{\prime}$, and note the point s; if es is equal to or less than one-half the thickness of the driver's tooth, the case is possible.

290. In gears that are likely to be used both as drivers and followers, the arcs of approach and recess are made nearly equal to each other, and in all cases the arc of action should be made greater than the pitch arc. Gears may be made with faces only, the companion gears having flanks only; in such cases the wheel having faces should always drive, in order to have the action receding, *i. e.*, they don't drive till by the pitch point, and the arc of recess should be greater than the pitch.

In FIG. 19 assume the same arc of approach and recess each one-half the pitch, and assume the teeth and spaces equal, there being no backlash, the case will just be possible with pointed teeth, if both $d_{\prime}i$ and es are one-fourth the pitch.

A practical example is shown in FIG. 20, which is lettered the same as FIG. 19. Here the wheel $O_{\prime}$ is the driver, and the path of contact is $D_{\prime}CE$; the whole arc of motion being $2\frac{5}{8}$ times the pitch and is divided in such a way that the arc of recess Cd is $\frac{1}{3}$ greater than the arc of approach, which is equal to $D_{\prime}C$ and se is less than $\frac{1}{2}$ the thickness of the driver's tooth.

Oct. 11th

Radial Flank Gearing.

[illegible]. As the hypocycloid generated by a circle rolling inside of another of twice its diameter is a radial line, it can be made use of in designing radial flank teeth, whose flanks, as the name implies, are radial.

Let $O_{\prime}$ and O (Fig. 21) be the centres of a pair of gear wheels, and let $AA_{\prime}$ and $BB_{\prime}$ be their pitch circles.

Then the flanks of the teeth of O, and the faces of the teeth of $O_{\prime}$ would be generated by the rolling circle CSO (having a diameter one-half that of $BB_{\prime}$) upon the inside of $BB_{\prime}$, and on the outside of $AA_{\prime}$. In the same way the flanks of the wheel $O_{\prime}$ and the faces of the wheel O are generated by the rolling circle $CS_{\prime}O_{\prime}$, whose diameter is one-half that of $AA_{\prime}$.

To find the proper length of the teeth, the arc of approach and recess being given: Lay off on the driver's pitch circle $AA_{\prime}$, $Ce =$ the arc of approach, and on $BB_{\prime}$, $Cd =$ the arc of recess. Draw the radial lines $eEO_{\prime}$ and dDO; then the path of contact is ECD. The arcs Ce and CE are in this case equal, since the angle $CO_{\prime}e$ is one-half $Co_{\prime}E$; for the same reason $dC = DC$.

The addendum circle of the follower O is drawn through E, and that of the driver $O_{\prime}$ through D. The flank circle, showing the acting flanks of the driver CG, is drawn through E about $O_{\prime}$; that showing the acting flanks of the follower CF is drawn through D about O, and the flanks of the teeth on each wheel are radii of the two circles.

Interchangeable Wheels.

292. When the flanks and the faces which act on each other are generated by the same describing circle, it is not necessary that the two describing circles be of the same diameter as, for example, the case of radial flank teeth; but if we desire to make a set of wheels, any two of which will gear, it is evident that we must use the same describing circle for all the faces and all the flanks. The question now arises, what shall be the diameter of this describing circle?

293. The size of the describing circle depends upon properties of the hypocycloid, which are shown in Figs. 22, 23, and 24. In Fig. 22, the describing circle is one-half the size of the pitch

circle, and the flanks are radial, which gives a comparatively weak tooth at the root. In Fig. 23, the describing circle is smaller, and the flank of the tooth curves away from the radius outwardly, thus giving a strong form of tooth. In Fig. 24 the describing circle is larger than in Fig. 22, and the flank curves inward, making a weak tooth, and one also difficult to cut.

294. From the above, the practical conclusion would appear to be that the diameter of the describing circle should not be more than one-half that of the pitch circle of the smallest wheel of the set. It will be found, however, that when the describing circle is taken at five-eighths of the diameter of the pitch circle, the curvature of the flanks will not be so great, nor with the ordinary proportions of height to thickness of teeth, that the spaces are any wider at the bottom than at the pitch circle; this being the case, the teeth can be shaped by a milling cutter.

295. The Pratt & Whitney Co., of Hartford, Conn., use the epicycloidal system, and make twelve the smallest number of teeth of any wheel; the diameter of the describing circle is taken such that it will give radial flanks on a fifteen-toothed pinion, or is five-eighths the diameter of the pitch circle for a twelve-tooth pinion, the smallest wheel of the set. This describing circle has been used with excellent results.

296. When laying out a single pair of wheels for which cutters are to be made expressly, very good results may be obtained for general work by using two describing circles, each equal to three-eighths that of the pitch circle in which it rolls. It is to be remembered also, that for a given arc of action, the face is shorter and the obliquity of the action less, the larger the describing circle. The reduction of the obliquity of action is sometimes of importance enough in very delicate mechanism, as watch or clock work of fine grade, to make it advisable to use teeth cut under like those of Fig. 24 as much strength as possible being obtained by filling up the corners *c*, only leaving space as required by the clearing curve of the engaging wheel. The parts thus filled in are usually termed "fillets".

Rack and Wheel.

297. A rack may be considered as an infinitely large wheel;

its pitch line would thus be straight, and the rack would travel through a distance equal to the circumference of the pitch circle of the wheel for every revolution of the latter. The pitch of the rack is found by rectifying the pitch arc of the wheel.

The construction of the rack tooth is shown in FIG. 25. The circles for forming the faces and flanks of the rack o and $o_{,}$ have here been taken the same; and it is clear that if the same circle be used for generating the faces and flanks of a set of wheels, any one of them will gear with the rack, provided the pitch is the same as that of the rack.

The faces and flanks of the rack teeth are cycloids, being generated by rolling a circle on a straight line.

If the addendum of the rack teeth vw, is given, and we consider the pinion to drive, action will begin at E where the addendum line vE, drawn parallel to $BB_{,}$, at a distance vw, cuts the describing circle o; if CE is given, the height vw is fixed.

The receding action ends at x, where the addendum circle of the pinion cuts the describing circle $o_{,}$. If now $Es_{,}$ is drawn perpendicular to BB, as in the preceding cases, the intercept on the pitch line to the right of it can not be greater than one-half of the thickness of the tooth, and should be less.

298. The case of a rack and wheel corresponding to that of radial flank gears is shown in FIG. 26. The describing circle, whose diameter is OC, equal to half the diameter of the pitch circle, generates the radial flanks of the wheel teeth and the cycloidal faces of the rack teeth. The diameter of the describing circle for the faces of the wheel and flanks of the rack is one-half that of the rack, which is infinite.* Hence we may consider $BB_{,}$, that circle, and the faces of the wheel become involutes of its pitch circle, while the flanks of the rack would become straight lines perpendicular to $BB_{,}$. The path of the contact to the left of OC is on the line $BB_{,}$, and the acting flank of the rack is merely a line, which is bad on account of liability of excessive wear.

The tooth path of contact is $ECD = gCd$.

The flank of the rack tooth may be made, as shown, an arc of a circle whose centre is on $BB_{,}$, and whose radius is equal to the radius of curvature of the wheel tooth at its highest point.

The radius may be found by drawing through N the highest point in the involute face, a tangent to the pitch circle AA_1, tangent at T, thus giving the radius NT. This curve somewhat strengthens the teeth, and also improves their receding action.

Annular Wheels.

299. Annular wheels are those having teeth cut on the inside of an annular rim. In this case one pitch circle lies within the other (FIG. 27), but the method of generating the tooth outlines and the nature of their action is the same as in FIG. 19 similar points being lettered the same in each figure. Two describing circles are used, each of which generates a face and its corresponding flank. The faces of the annular wheel are inside, and the flanks outside, of its pitch circle; it is also to be noticed that the contour of the annular wheel, so far as the acting tooth curves are concerned, is the same as that of the spur wheel, having the same pitch and describing circles, the tooth of one corresponding to the space cf the other.

In the figure the describing circles were taken of different diameters, but they might have been the same, and in that case the annular wheel would be interchangeable with spur wheels for which the same describing circle was used, within certain limits.

Diametrical Pitch.

300. In designing spur gearing it is necessary to find the necessary pitch in order to lay out the tooth shape, and also to determine its strength. It would be more convenient to express the pitch by whole numbers or manageable fractions, as 2″ pitch, ¾″ pitch, etc., but since the circumference is π times the diameter, awkward decimals will be obtained in the diameter. Practically it is much more important to have the diameter an even number rather then the circular pitch, provided the latter gives a tooth which is strong enough. This has led to the introduction for almost all cut gearing, of what is called *diametrical pitch;* which is simply the quotient found by dividing the diameter of the pitch circle by the number of teeth the wheel is to have; whence,

$$\text{Diametrical pitch} = \frac{\text{Diameter}}{\text{Number of teeth}} = \frac{\text{Circular pitch}}{\pi}$$

$$\text{Circular pitch} = \text{Diametrical pitch} \times \pi$$

301. In using this system convenient values of the diametrical pitch are selected, each being a fraction having unity for its numerator and an integer for its denominator. as 1, ½, ¾, ⅜.

The pitches are commonly given by using the denominator only ; thus an "8-pitch wheel" is one which has eight teeth for each inch of diameter, or whose diameter pitch is ⅛. By this system the diameter can be calculated very easily.

Required the diameter of a 4-pitch wheel with 37 teeth.

$$\frac{87}{4} = 9\tfrac{1}{4}'' = \text{Diameter.}$$

How many teeth of 16-pitch has a wheel 3⅞″ diameter?

$$3\tfrac{7}{8} \times 16 = 62 = \text{Number of teeth.}$$

The diametrical pitch is sometimes defined as the reciprocal of the above, and in such case the numerator is used, thus: Diametrical pitch is the number of teeth to one inch of diameter,

$$\frac{\text{Number of teeth}}{\text{Diameter}} = \text{Diametrical pitch, or. diametral pitch.}$$

The term diametrical pitch is also applied to racks: thus a rack of 8-pitch is one that will gear with an 8-pitch wheel.

Arbitrary Proportions.

302. The teeth of gear wheels may be designed by the preceding methods so as to fulfill any proposed conditions of approaching and receding action. In a majority of cases the exact lengths of the approaching and receding action are not important provided they are long enough. It is a very common practice to make the whole height of a tooth a certain fraction of the pitch; the part which projects outside of the pitch circle being made a little less than that within, to allow the proper clearance..

None of the arbitrary proportions can be considered absolute, as the proper amount of clearance and backlash depend on the precision with which the tooth outlines are laid out to begin with, and then on the accuracy with which the teeth are made to conform to these outlines.

In the best cut gears manufactured to day the teeth barely clear each other when the fronts are in contact, and in any case the allowance made should depend on the accuracy of the workmanship. In cast gears more clearance is necessary to allow for irregular shrinkage or for light derangements of the mould. Axes must be carefully spaced in the epicycloidal system, in order that the teeth may work on their given pitch circles.

The following table gives some of the proportions in common use. The letters in the first column refer to FIG. 1, and P denotes the circular pitch.

	1	2	3	4
Length 1	$\frac{7}{10} P$	$0.67\ P$	$\frac{4}{5}\ P$	$0.750\ P$
Clearance . $d - h = ab$	$\frac{1}{10} P$	$0.07\ P$	$\frac{1}{15}\ P$	$0.060\ P + 0.04''$
Working depth . . . $2h$	$\frac{6}{10} P$	$0.60\ P$	$\frac{11}{15}\ P$	$0.690\ P - 0.04''$
Addendum . . . . . . h	$\frac{3}{10} P$	$0.30\ P$	$\frac{11}{30}\ P$	$0.345\ P - 0.02''$
Thickness . . . . . CF	$\frac{5}{11} P$	$0.475\ P$	$\frac{7}{15}\ P$	$0.470\ P - 0.02''$
Space . . . . EF	$\frac{6}{11} P$	$0.525\ P$	$\frac{8}{15}\ P$	$0.530\ P + 0.02''$
Backlash . . . $EF - CF$	$\frac{1}{11} P$	$0.05\ P$	$\frac{1}{15}\ P$	$0.060\ P + 0.04''$

In the first three systems the percentage of backlash is constant, the backlash thus increases with the pitch. It is better, however, to decrease the percentage of the backlash as the pitch grows larger; for the greater the pitch the smaller will be the proportion borne to it by any unavoidable error in workmanship. The last system (4) of Fairbairn and Rankine is based on this view of the proper proportions of backlash, the amount of backlash given is, however, much larger than that now generally allowed.

Teeth proportioned by any of the four systems will in general be of good shape, and answer their purpose. Should the wheel have less than twelve teeth, or should the exact amount of approaching or receding action be of importance, no arbitrary system is to be used, but the proper dimensions should be determined by the methods as explained. The backlash and clearance in any case should always be as small as the quality of workmanship will permit.

Since theoretically the teeth can be in contact on both sides at once, the backlash has been disregarded in the diagram. Should it be introduced, the thickness of the tooth, instead of being one-half the pitch, would be one-half the pitch minus one-half the backlash.

303. PROPORTIONS FOR DIAMETRAL PITCH. In using the diametral pitch the working depth of the tooth is usually taken as twice the diametral pitch; the clearance is then commonly taken as one-eighth the diametral pitch, thus making the length of the tooth 2 times the diametral pitch. This system is used by the Pratt and Whitney Co. In an "eight-pitch" wheel the working depth is $\frac{2}{8}'' = \frac{1}{4}''$, the addendum is $\frac{1}{8}''$, and the clearance is $\frac{1}{8} \times \frac{1}{4} = \frac{1}{32}''$. The diameter of the gear blank, or the addendum circle, could easily be found by adding two to the number of teeth on the wheel and then multiplying by the diametral pitch. Thus the diameter for a 60 tooth, 8-pitch wheel blank would be

$$60 + 2 \times \frac{1}{8} = \frac{60 + 2}{8} = 7\tfrac{3}{4}''.$$

$$\text{Diameter of pitch } C = \frac{\text{Number of teeth}}{D\,P}$$

Spur Gearing—Involute System.

304. Next to the epicycloid the curve most extensively used as the tooth outline in spur gearing is the involute of the circle. Of late the involute seems to be coming more and more into favor.

Let O and O_1 (FIG. 28) be the centres and C the pitch point of a pair of pitch circles AA_1 and BB_1. Through C draw the line TT tangent to the pitch circles, and the line FCG making an angle TCF with TT; this angle is called the *angle of obliquity*.

Upon the line FCG drop perpendiculars Oc and O_1d from O and O_1 respectively, and with these perpendiculars as radii draw the circles DD_1 and EE_1, known as the *base circles*.

Now suppose the base circles to be pulleys connected by an inextensible cord cd; then the angular velocity ratio will be,

$$\frac{\text{Angular velocity } O}{\text{Angular velocity } O_1} = \frac{O_1d}{Oc}$$

But for the similar triangles OCc and O_1Cd we have

$$\frac{O_1d}{Oc} = \frac{O_1C}{OC} \text{ or } \frac{\text{Angular velocity } O}{\text{Angular velocity } O_1} = \frac{O_1C}{OC}$$

That is, the angular velocity ratio of the pulleys is the same as that of the pitch circles from which they were derived. Let the upper pulley *DD* turn in the direction shown by the arrow; it will wind the cord *dc* upon itself and unwind it from $EE_{\prime}$. If a point, as *d*, upon *dc*, act as a describing point, it will trace upon the plane of the circle *O* an involute *da* of the base circle $DD_{\prime}$, and on the plane of the circle $O_{\prime}$ an involute *dg* of the base circle $EE_{\prime}$. As these two curves *da* and *dg*, in contact at *d*, were simultaneously generated by a point *d* of the line *dc*, they will be in contact along *dc*, and the common normal to both curves, *dc*, will always pass through the pitch point *C*.

These curves are, therefore, proper ones for a pair of teeth, and form both face and flank. The path of contact is the straight line *dc*, and the angle of obliquity is *TCd*.

305. If the centres *O* and $O_{\prime}$ be made to approach or recede from one another, the path of contact *dc*, which is the tangent to the base circles, will change its obliquity, but the ratio of $O_{\prime}C$ and *OC*, the segments of the line of centres, will remain unchanged. Also the curves *da* and *dg*, being involutes of the base circles, will not be affected by the change. Involute teeth have consequently the important and peculiar property of permitting the distance between centres to be changed, within limits, without destroying the accuracy of action of the teeth.

306. Normal pitch. The normal pitch is the distance on the path of contact, or on any common normal, from the face of one tooth to the face of the next tooth. From the properties of the involute it is the same as the distance from the front of one tooth to the front of the next on the base circle; and from the method of generating the tooth outline it might be defined, as being equal to the length of the line of connection that passes the pitch point while the pitch passes it.

Let *P* be the circular pitch, *n* the number of teeth of a wheel, ϕ the angle of obliquity, and let *N* be the normal pitch; then (Fig. 29)

$$P = \frac{2\pi OC}{n}, \text{ or } N = \frac{2\pi Og}{n} = \frac{2\pi OC \cos\phi}{n} \ \therefore\ N = P \text{ cas } \phi$$

This might be derived in another way, thus:

$$\frac{N}{P} = \frac{Og}{OC} = \cos\phi, \text{ whence } N = P\cos\phi.$$

307. Since the curve, which forms the face and flank of a tooth in this form of gearing, depends only on the diameter of the wheel and the obliquity of action, wheels may be designed separately by keeping the normal pitch of each wheel of a set the same.

Let O (FIG. 29) be the centre and C the pitch point of the wheel, and assume the obliquity of action to be $\psi = 14\frac{1}{4}^\circ$.

1. TO DRAW THE PATH OF CONTACT AND THE BASE CIRCLE.

Draw TC tangent to the pitch circle at the pitch point; lay off the angle $TCd = \psi$ and draw dCg. This is the path of contact, and is the common tangent to the base circle of the given wheel and of the wheel with which it will gear. Draw Og perpendicular to dCg; then Og is the radius of the base circle $GG_{\prime}$. The two angles dCT and COg are equal, since their sides are perpendicular to each other; consequently Og may be calculated by the formula

$$Og = OC \cos \psi.$$

308. 2. TO FIND THE ADDENDUM, AND TO DRAW THE ADDENDUM AND ROOT CIRCLES.

In order to insure the contact of at least two pairs of teeth the arc of contact should be twice the pitch, or the path of contact should be twice the normal pitch; hence lay off $Cc = Cd$, the normal pitch, and dc will be the path of contact. The addendum is the distance da, measured on a radial line, from d to the pitch circle through c, about the centre O.

To draw the root circle, supposing that this gear must work with any one of a set having the same path of contact dc, it is necessary to find the addendum of that wheel having the same pitch, which has the longest teeth, or, what amounts to the same thing, of that wheel which has the fewest teeth.

A wheel which has 25 teeth is commonly taken to be the smallest wheel for involute gearing; in it the pitch subtends an angle of $14\frac{1}{4}^\circ$ nearly, and from the previous reasoning we may take the angle of obliquity $14\frac{1}{4}^\circ$, which has been found to work well in practice.

309. The construction of a rack to work with involute teeth may be considered to be a limiting case of involute gears. If the diameter of the wheel increase indefinitely, the limit of the pitch

circle will be the pitch line $AA_{\prime}$, Fig. 30, and the limit of the involute face of the tooth will be a line perpendicular to the path of contact.

To construct the outline of a rack tooth to work with the wheels of Fig. 29, draw the path of contact dCc Fig. 30 through the pitch point C, making the angle $dCT = \phi$; and lay off Cd and Cc each equal to the normal pitch of the wheel with which the rack is to work. dn parallel to the pitch line is the addendum line, and ex is the flank line. Draw Cr perpendicular to the pitch line and equal to the greatest addendum of the set of wheels having the given pitch, plus the clearance; then $RR_{\prime}$ is the root line.

310. Rankine's rule to make the entire length of the tooth 0.75 P, or that of Willis to make it 0.7 P, will give good results, experience has shown that for ordinary purposes the obliquity should not exceed from 15° to 17°. When it is no greater than this, it is safe to say that these teeth are equal to the epicycloidal or any others, even for heavy work; in addition to other advantages that shall be mentioned, it is to be observed that the form of the tooth is a strong one.

311. Peculiarities of involute teeth.

1. Sets of wheels with involute teeth have a definite normal pitch equal to the pitch measured on the base circle.

2. All wheels and racks with involute teeth of the same normal pitch, gear correctly with each other.

3 The distance between the centres of the wheels may vary without affecting the velocity ratio, and hence the backlash may be adjusted.

By the application of the preceding principles two or more wheels of different numbers of teeth, turning about one axis, can be made to gear correctly with one wheel or one rack; or two or more parallel racks with different obliquities of action may be made to gear correctly with one wheel, the normal pitches in each case being the same. Thus differential movements may be obtained which are not possible with teeth of any other form.

The principal objection to the use of involute teeth for large gears is the great obliquity of action and the large number of teeth in the smallest wheel.

On the other hand, for smooth action, especially for light work, it may be an advantage to have this constant obliquity of action; as the side pressure will tend always to keep the axes at the greatest possible distance from each other, thus preventing jarring in case there may be any looseness in the bearings.

312. ANNULAR WHEELS WITH INVOLUTE TEETH. The construction is shown in FIG. 31, O and $O_{\prime}$ being the centres of the annular wheel and pinion respectively. The constant angle of obliquity is TCF, and the radii of the base circles are found by dropping the perpendiculars Oc and $O_{\prime}d_{\prime}$. The construction and lettering is the same as in the previous figures. If the path of contact is taken as cd, the addendum circle for the wheel is drawn through c, and that of the pinion through d. It is to be observed that there is very little sliding in this case, due to the small difference in length of the acting curves. Since, in any case, action may begin or end at $d_{\prime}$, the greatest possible length of the wheel tooth is found by drawing the addendum circle therefor through the point $d_{\prime}$ about O. This form of tooth would appear to be well adapted for inside gearing where smoothness of action was required, on account of the constant obliquity and small sliding action.

Pin Gearing.

313. In this form of gearing the teeth of one wheel consist of cylindrical pins, and those of the other of surfaces parallel to epicycloidal surfaces, from which they are derived.

In FIG. 32 let O and $O_{\prime}$ be the centres of the pitch circles whose circumferences are divided into equal parts, as CE and CG. Now, if we suppose the wheels to turn on their axes, and to be in rolling contact at C, the point E of the wheel O will trace the epicycloid GP on the plane of the wheel $O_{\prime}$, and merely a point E upon the plane of O. Let CF be a curve similar to GE, and imagine a pin of no sensible diameter, a rigid material line, to be fixed at C in the upper wheel. Then if the lower one turn to the right, it will drive the pin before it with a constant velocity ratio, the action ending at E if the driving curve be determined at F as shown.

If the pins be made of a sensible diameter, the outlines of the teeth upon the other wheel are curves parallel to the original epicycloids, as shown in FIG. 33. The diameter of the pins is usually made about equal to the thickness of the tooth, the radius being therefore, about ¾ the pitch arc. This rule is, however, not imperative, as the pins are often made considerable smaller.

Clearance for the pins is provided by forming the root of the tooth with a semi-circle of a radius equal to that of the pin, the centre being inside of the pitch circle, an amount equal to the clearance required.

The pins are ordinarily supported at each end, two disks being fixed upon the shaft for the purpose, as shown in FIG. 7, PLATE VIII, thus making what is called a lantern wheel or pinion.

314. MODE OF ACTION. In wheel work of this kind the action is almost wholly confined to one side of the line of centres. In the elementary form (FIG. 32) the action is wholly on one side, and receding, since it cannot begin until the pin C is reached (if $O_{,}$ drives), and ceases at E; if O is considered the driver, action begins at E, ends at C, and is wholly approaching. As approaching action is injurious, pin gearing is not adapted for use where the same wheel has both to drive and follow; the pins are therefore always given to the follower, and the teeth to the driver.

315. RACK AND WHEEL. As the pins are always given to the follower, we have two cases:

1. *Rack drives*, giving the pin wheel and rack, FIG. 34. Here the original tooth is bounded by cycloids generated by the pitch circle of the wheel.

2. *Wheel drives*, giving the pin rack and wheel. Here (FIG. 35) the original tooth outline is the involute of the wheel's pitch circle.

316. INSIDE PIN GEARING. Here also there are two cases.

1. *Pinion drives* (FIG. 36). The original tooth outlines will be internal epicycloids generated by rolling the pitch circle of the annular wheel on the pinion's pitch circle.

2. *Annular wheel drives* (FIG. 37). Here the original tooth outline is the hypocycloid traced by rolling the pinion's pitch circle in the wheel's circle.

317. PATH OF CONTACT. In the elementary form of tooth (FIG. 32) the path of contact is on the circumference of the pitch circle of the follower *O*, as *CE*. When a pin is used, its centre always lies in this circumference, and its path of contact may be found by laying off a distance *EI*, equal to the radius of the pin (FIG. 38) on the common normal. Drawing a number of these common normals, all of which must pass through the pitch point *C*, and laying off the radius of the pin *EI* on each, we have the path of contact *CI*.

Double Point Gearing.

318. This form of gearing, shown in FIG. 39, gives very smooth action where not much force is to be transmitted. The pitch circles are here taken as the describing circles; the face *Cg* of the pinion *O* is generated by rolling the pitch circle $O_{,}$ on that of *O*; and the face *CF* is generated by rolling the pitch circle *O* on $O_{,}$. If *O* is considered the driver, action begins at *D*, the point *C* of *O* sliding down the face *fC* while *C* travels from *D* to *C*. In the receding action the point *C* of the tooth $O_{,}$ is acted on by the face *Cg*, while *C* moves from *C* to *E*. The spaces must be made so as to clear the teeth. This combination reduces friction to a minimum and gives the obliquity of action less than in any case except pin gearing, but the teeth are much undercut and weakened by the clearing curves, and if much force is to be transmitted, the line of contact will soon be worn away.

319. SHROUDED WHEELS. When the teeth of a wheel as *O* (FIG. 39) are undercut and weak, they are sometimes united at their ends by annular rings cast with the wheel, and the wheel is then said to be shrouded. This shrouding strengthens the teeth and is usually applied to the pinion; where the wear is greater, it may extend the whole depth of the teeth of the pinion, or both pinion and wheel may be shrouded to half the length of the teeth. The latter arrangement is seldom adapted on account of the difficulty of casting the wheels.

Sang's Theory.

320. A conjugate curve has already been defined, (§ 273). If in FIG. 40 we assume any tooth outline as $a_{,}Ca$ we may construct

a series of wheels, assuming different pitch circles, that will work with the wheel having the assumed tooth outline. Then any one of the series may be taken and a second series of wheels made from its tooth outline. From the method of generating the tooth outlines, it will follow that any wheel of the first series will gear with any one of the second. Now if the conjugate tooth outlines on any two equal wheels be made the same, the two series will be identical and the wheels will all be interchangeable. If the tooth and space are taken equal to each other, two racks formed from the above equal wheels will exactly fit into each other, the tooth of one filling the space of the other and being at the same time equal to that of the other.

Hence we may assume the outlines of a rack tooth such that it is made up of four equal lines in alternate reversion (thus giving a symmetrical tooth about its centre line) and from it derive an interchangeable set of wheels. If the rack be made up of equal cycloidal arcs, the epicycloidal system with a constant describing circle will be obtained ; if it is bounded by oblique straight lines the involute system will be produced.

The rack may be arbitrarily assumed, provided it fulfills the above conditions, and different series of wheels may be derived. This method of constructing the tooth outline, which is practically the reversal of the ordinary way, is due to *Professor Sang*.

Should we now shape a cutter to the exact outline of a rack tooth and then give to it a reciprocating motion, parallel always to the axis of a gear-wheel blank, which is made to move in reference to the rack tooth just the same as it would if the pitch surface of the rack and that of the wheel were in rolling contact with each other, the rack tooth would shape the space of the wheel as shown in FIG. 41, where several positions are given. It being understood that the wheel is not moved while the rack tooth is cutting, and that it is only rolled fast enough to give a light chip to cut each time the tooth moves in the cutting direction. One space being cut the blank is turned through the pitch angle and the operation is repeated until the wheel is formed. This method has been applied in a bevel gear cutting engine.

Unsymmetrical Teeth.

321. In all the figures hitherto given, the teeth are symmetrical, so that they will act equally well whether the wheels are turned one way or the other. In cases where the action is always one way it may be advantageous to make the teeth otherwise, as shown in FIG. 42. Here the lower wheel $O_{\prime}$ is the driver and the acting outlines of both wheels are of epicycloidal form ; the describing circle o and $o_{\prime}$ have been taken large to reduce the obliquity to a minimum. If the other sides of the teeth were made the same we would have a weak tooth at the root. To avoid this the backs of the teeth may be made involutes of considerable obliquity, the radii of the base circles being Oc and $O_{\prime}d$. It can be seen that this gives a very strong form of tooth.

Twisted Gearing.

322. HOOKE'S STEPPED WHEELS. If a pair of spur wheels are cut transversely into a number of plates, and each plate is rotated through an angle, equal to the pitch angle divided by the number of plates, ahead of the adjacent plate, as shown in FIG. 43 we will have a pair of stepped wheels, first introduced by Dr. Hooke. By this device we obtain the effect of increasing the number of teeth without diminishing their strength ; the number of contact points is also increased, and the interval between their times of crossing the line of centres, where the action is best, is correspondingly diminished. The upper figure shows a section on the pitch line $AA_{\prime}$. The action for each pair of plates is the same as that for spur wheels having the same outlines. In practice there is a limit to the reduction in the thickness of the plates depending on the material of the teeth and the pressure to be transmitted, since too thin plates would abrate. The number of divisions is not often taken more than two or three and the teeth are thus quite broad. These wheels give a very smooth and quiet action.

323. HOOKE'S SPIRAL GEARING OR TWISTED GEARING. If the number of plates be taken infinite the effect is the same as that explained in (§ 266, 4°). The twisting being uniform, the elements of the teeth become helices, all having the same pitch. The line of contact between two teeth will have a helical form but will not be a true helix ; the projection of the helix on a plane

perpendicular to the axis will be the ordinary path of contact. It can easily be seen that the common normal at any point of contact can in no case lie in the plane of rotation but will make an angle with it. The line of action then can in general have three components :—

1. A component producing rotation perpendicular to the plane of the axes.
2. A component of side pressure, parallel to the line of centres.
3. A component of end pressure parallel to the axes.

When the point of contact is in the plane of the axes the second component is zero; advantage may be taken of this, so that there may be no sliding action between the teeth. The end pressure is neutralized as explained (see § 266).

324. SLIDING FRICTION ELIMINATED. In this case the angle of twist is at least equal to the pitch angle and should be a little more. In FIG. 44, which represents a transverse section of a pair of twisted wheels, suppose the original tooth outlines to have been those shown dotted. Then cut away the faces as shown by full lines having the new faces tangent to the old ones at the pitch point *C*, we now have lost proper contact except that at *C* for the section shown, but by twisting the wheels this contact can be made to travel along the common element of the pitch cylinders through *C* from one side of the wheel to the other. A simple construction to use in this case is to make the flanks of the wheels radial and the faces semi-circles tangent to the flanks. The action here is purely rolling and is very smooth and noiseless, but for heavy work it is best to use the common forms of teeth with sliding action, so that the pressure may be distributed over a line instead of acting at a point.

Approximate Forms of Teeth.

325. To secure perfect smoothness of action in toothed wheels it is necessary that the tooth outlines should be accurately laid out, as explained in the preceding pages, and that the teeth when constructed should conform exactly with the outlines found. If the teeth are to be cut, the exact curves should be used, as when the cutter is once made it will cut the accurate shape as well as

any other. When, however, the teeth are to be cast, or for some other reason perfect accuracy is not required, the exact curves may be replaced by others which approximate to them more or less closely, but which are similar to construct. This is possible as, the teeth being short, only a small part of the theoretical curve is used. In these approximations the proportions of the teeth are usually governed by one of the sets of arbitrary proportions given in par. 302. The two principle methods of approximation are

1°. By circular arcs,
2°. By curved templates.

326. ROBINSON'S TEMPLATE ODONTOGRAPH. This ingenious instrument, the invention of Professor S. W. Robinson, gives the outline of the tooth direct, and may be used in the pattern shop for laying out gear patterns. It was found that the curve, to satisfy the mathematical conditions,

1st, must be one of rapidly changing curvature, approximating very closely to the epicycloid;

2nd, it must be very nearly perpendicular to the pitch circle at the middle point of the tooth outline;

3rd, it must intersect the addendum circle at the same point as the epicycloid, in short, it must coincide with the epicycloidal face. The curve most completely satisfying these conditions was found to be a logarithmic spiral.

The odontograph consists of a thin brass plate *ABF* (FIG. 45), graduated on the edge *AB*, the figure showing the instrument about one-sixth size. Accompanying the instrument are tables varying according to the kind of tooth required. FIG. 45 shows the method of using the odontograph to lay out a wheel belonging to an interchangeable series. The tables are arranged in four columns, giving

1, Diameter in inches,
2, Number of teeth,
3, Face settings,
4, Flank settings.

Let LCL_1 be the pitch circle, which is known when the pitch and the number of teeth are given, assume *C* the middle point of a tooth, and lay off the arc *CE* its half thickness. Draw tangents *CT* and *ES* to the pitch circle at *C* and *E*. Set the instrument in the position *ABF*, the proper division on the scale, found from the column of face settings, being brought to *D*, while at the same time the curved edge *AF* is tangent to *CT*, and *E* is on the edge *AB*; now draw the face *Ea*. To draw the flank the instrument is placed in the position $A_1B_1F_1$, the proper flank reading being at *E*, and the curve *AF* being tangent to *FS*. Then draw the flank *Eb*, the addendum and root circles are then drawn as indicated, using arbitrary proportions.

327. *Professor J. F. Klein* has recently introduced a method of finding correct tooth outlines by means of tables especially prepared for the different systems of gearing. The system consists in giving, by table, the distance of points of the tooth outline from each of two perpendicular reference lines *XX* and *YY* (Fig. 46) drawn through some easily fixed point in the tooth outline. Each of the two sets of distance is expressed in simple decimal fractions of the pitch or diameter, and, for ease in tabulation computation and laying out, one of these sets is arranged in groups of equidifferent values. The computations from the tables are very simple, only short division and multiplication being nesessary for determining ordinary outlines. After making the computations, the drawing of the outline only involves the use of the square and the ability to lay off distances accurately. Tables are also given by means of which, two reference lines, one on each side of a tooth, may be located in proper relation to each other, thus making it unnecessary to use compasses. This method is especially useful in laying out the outlines of the teeth of large wheels where it would be inconvenient to roll up the curves in the ordinary way.

Patent Cutters.

328. Patent cutters are manufactured for the teeth of gear wheels, which, from their peculiar construction, can be sharpened when dull, by grinding the faces of the teeth. This operation can be repeated without altering the form of the tooth which the cutter makes, thereby rendering them many times more valuable than cutters of ordinary form.

Orders should be given by tables, stating the number of cutter and the diametral pitch required.

In ordering cutters for beveled wheels give the largest pitch in diameter, the number of teeth in each wheel, and the length of face of teeth. In ordering cutters for worm wheels, give the number of teeth in wheel, the diameter of worm and the number of threads to the inch.

329. DIRECTION FOR USING THE CUTTERS. The cutters should be kept perfectly sharp by grinding the face of the teeth on the side of a solid emery or vulcanite wheel, which has its edge beveled on one side so as to reach the bottom of the teeth. This wheel should be put on an arbor with a shoulder and nut, so that the flat side will run true, and at a velocity of from 2000 to 3000 revolutions per minute. If used in a common hand lathe the top of the rest should be made square or vertical to the face of the wheel, or what is better, use a small platform in the place of the rest. The cutters should not be crowded too hard, especially when cutting through at the end of a tooth. The depth of the space made by the cutters affords simple clearance, as it exceeds the working depth of the tooth by an amount equal to one-tenth of the thickness of the tooth on the pitch line.

The Sizing and Cutting of Gear Wheels.

330. The "pitch diameter" is two parts of the pitch less than the whole diameter. For instance, a wheel containing 30 teeth of ten pitch, would be three inches in diameter on the pitch circle, and $3\frac{2}{10}$ in outside diameter.

The term "circular pitch" is used to denote the distance on the pitch circle from the centre of one tooth to the centre of the next tooth.

Calculations of diameters and teeth of wheels are more simple in obtaining diametral than circular pitch, and the facility of more readily acquiring the necessary knowledge required to make these calculations, is leading to the more general use of this method.

331. To facilitate the measurement of wheels to be sized according to diametral pitch, either of the following steel rules described, can be used. The 1st is a 12 inch rule containing four lines of graduations upon each side, one each as follows, 18ths, 20ths, 22nds, 24ths, 26ths, 28ths, 30ths, 32nds. Each line of

graduations is figured the whole length of the rule 10, 20, 30, etc. Suppose a wheel of 60 teeth of the 20 pitch to be sized, then find 60 on the line of 20ths, and that will be the pitch diameter of the required wheel; then add two of the divisions to make the outside diameter which will be sixty-two twentieths. The 2nd is also a 12 inch rule having cne inch only of graduations on each end as follows: 6ths, 7ths, 8ths, 9ths, 10ths, 11ths, 12ths, 14ths, 16ths, 18ths, 20ths, 22nds, 24ths, 26ths, 28ths, 30ths, 32nds, 34ths, 36ths, 38ths. The intermediate ten inches are blank, except that the inch lines are made clear across the rule. Suppose a wheel of 83 teeth of 10 pitch is to be sized, then take eight of the blank inches and three of the tenth graduations and that will give the pitch diameter of the required wheel, add two of the tenths which will give the outside diameter which will be eight and five-tenth inches.

This rule furnishes graduations for a large variety of pitches and is the best adapted for the use, for which it is designed.

According to the system adopted by the *Brown & Sharpe Mfg. Co.*, any wheel of one pitch will gear into any other wheel or into a rack of the same pitch. *Eight cutters are required for each pitch.* These eight cutters are adapted to cut from a pinion of twelve teeth, to a rack, and are numbered respectively, 1, 2, 3, etc. The number of teeth and the pitch for which a cutter is adapted is also marked on each.

No. 1	will	cut	wheels	from	135	teeth	to a rack		
No. 2	"	"	"	"	55	"	to	134	teeth,
No. 3	"	"	"	"	35	"	"	54	"
No. 4	"	"	"	"	26	"	"	34	"
No. 5	"	"	"	"	21	"	"	25	"
No. 6	"	"	"	"	17	"	"	20	"
No. 7	"	"	"	"	14	"	"	16	"
No. 8	"	"	"	"	12	"	"	13	"

If a cutter is wanted for a wheel of forty (40) teeth, of 8 pitch, then the cutter required, would be No. 3 of 8 pitch, inasmuch as No. 3 cutter will cut all wheels containing from 35 to 54 teeth, inclusive. It should be borne in mind that eight different cutters are required in order to cut all the wheels of any given pitch.

As these cutters allow of being ground when dull, it is important that they be kept sharp.

It is desirable in applying gearing of any kind, to avoid having wheels or pinions with a small number of teeth. Pinions of twelve teeth will work very well but a less number of teeth should not be used.

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Few mechanics are familiar with the minutiae of gearing, and the necessity of exact sizing of wheels, as to diameter, is often overlooked. Special care is also required to know that the distance or the centres of two wheels running together is correct relatively to the diameters.

Bevel Wheels.

332. The curve of teeth in bevel wheels, when correctly formed, changes constantly from one end of the tooth to the other. Therefore bevel wheels, whose teeth are produced with a cutter of fixed curve, are not theoretically correct, the cutter usually being of a curve that will make the correct form at the outer part of the face of the wheel, and of necessity leaves the curves too large at the inside ends of the teeth. Small bevel gearing is almost universally produced in this manner, which practically answers the purpose, except when the teeth are very coarse or the wheels very small, in which cases their operation is not satisfactory. For instance, a bevel pinion with less than 25 teeth cannot be cut with a cutter of fixed curve so as to work, except by repeated operations of cutting, for the purpose of giving the required curve on the inside end of the teeth. In place of cutting by changing position of the cutter, etc., the teeth are often filed slightly in order to round them off to the curve required for their free running. On all bevel wheels cut with a cutter of fixed curve, it is necessary to cut through twice, owing to the necessity of making the thickness of the cutter on the pitch line equal to about 0.005″ thinner than the space between the teeth at the smallest pitch diameter. As the width of space between the teeth on the largest pitch diameter should be greater than the thickness of the cutter, it must be made by passing the cutter through the second time.

333. When a pair of bevel wheels are of the same size and number of teeth, with their lines of centres at right angles, they are called "Mitre Wheels", and one cutter, of course, will answer for both; but where one wheel has a greater number of teeth, or differs in bevel from the one running into it, then each of the pair of wheels requires a different cutter.

The End

PLATES XVII AND XVIII.

Trains.

334. A train in mechanism consists of a series of moving pieces, each of which is follower to that piece which drives it, and driver to that which follows it. In the case of a train of elementary combinations, the comparative motion of the last follower and first driver is found by multiplying together all the velocity ratios of the several elementary combinations of which the train consists, each ratio having the directional relation with which it is connected denoted by means of the positive or negative algebraic sign, as the case may be. This product is the velocity ratio of the last follower and first driver, and their directional relation is indicated by the algebraic sign of that product.

335. Motion in one direction being positive, and in the opposite direction being negative, the last follower and first driver, move in the same or opposite directions, according as this product of the velocity ratios of the elementary combinations of which the train consists has the positive or negative algebraic sign.

336. A train of wheelwork consists of axes, each having upon it two wheels, one of which is driven by a wheel on the preceding axis, while the other drives a wheel on the following axis. If the wheels are all in outside gearing or external contact, or connected by a crossed belt, the direction of rotation of each axis is contrary to that of the adjoining axes. In some cases a single wheel upon one axis answers the purpose both of receiving motion from a wheel on the preceding axis and giving motion to a wheel on the following axis. Such a wheel is called *an idle wheel* it affects the direction of rotation only, and not the velocity ratio. A wheel and pinion upon the same axis is the mechanical equivalent of a lever with unequal arms, and modifies the force which may be tranmitted through it, while a single wheel on an axis is the equivalent of a lever with equal arms, and produces no modification in the intensity of the force which it may transmit.

337. So when any number of separate wheels on separate axes, are in gear, or in external contact one with the other, they are

equivalent to a single pair of wheels, the first wheel and the last wheel. The intermediate wheels act as carriers only, and transmit the motion through the intervening space. Thus if we have the wheels A, B, C, D, K, L, in external contact one with the other, and on separate axes, the velocity ratio of the first and last wheel equals the product of the velocity ratios of the elementary combinations that constitute the train. If we call this ratio E, we have

$$E=\frac{A}{B}\times\frac{B}{C}\times\frac{C}{D}\times\frac{D}{K}\times\frac{K}{L}=\frac{A}{L}$$

338. A belt and a pair of pulleys supply a mechanical equivalent for spur wheels, the belt may be open or crossed, and in either case the number of revolutions of the first wheel is to the revolutions of the second wheel as the diameter of the second is to the diameter of the first. The crossing of the belt merely reverses the direction of rotation of one of the pulleys and therein is the equivalent of inserting or removing an idle wheel; whence it follows that two pulleys and a crossed belt are equivalent to two spur wheels in gear, and two pulleys and an open belt are equivalent to three spur wheels in gear.

339. In the preceding chapters, the elementary combinations discussed have, in general, consisted of two principle parts only, a driver and follower; and it has been shown how to connect them to produce the required velocity ratio.

There may occur cases, however, in which, although theoretically possible, it may be practically inconvenient, or even impossible, to effect the required combinations of motions by a single combination. In such a case a series or train of combinations is employed, in which the follower of the first combination of the train is carried by the same axis or sliding piece to which the driver of the second is attached; the follower of the second is similarly connected to the driver of the third and so on. In this connection revolving pieces only, or *trains of wheels*, will be discussed.

340. FIG. 1, PLATE XVII, shows an example of a train of wheels where the follower B and the driver C are placed on the same axis, A being the first driver. The second follower D, and the

third driver E, are also on the same axis. The numbers preceding the letters t indicate the numbers of teeth on the wheels upon which they are placed.

341. VALUE OF A TRAIN OF WHEELS. By the value of a train we mean the ratio of the angular velocities of the first and last axes, or, what is equivalent, the ratio of the synchronal rotations. Let n_1 represent the number of rotations of the first or driving axis of a train, and n_m the rotations of the last axis in the same train; then if we designate by e the value of the train, we have

$$e = \frac{n_m}{n_1}.$$

The value of e is regarded as positive or negative according as the first and last axes revolve in the same or opposite directions.

Let a train of wheels like that in FIG. 1 have m axes numbered from 1 to m, and let n_1, n_2, n_3, n_4 . . . n_m represent the number of rotations of the different axes in the same time, then we have

$$e = \frac{n_m}{n_1} = \frac{n_2}{n_1} \times \frac{n_3}{n_2} \times \frac{n_4}{n_3} \times \ldots \times \frac{n_m}{n_{m-1}}$$

That is, the value of the train may be found by multiplying together the separate ratios of the synchronal rotations of the successive pairs of axes.

342. The comparative rotation of wheels is estimated in various ways: thus, let n_1 be the number of rotations made in a given time by the driver of a pair of wheels in gear, D its diameter, R its radius, T its number of teeth, P its period of time of one rotation, and V its angular velocity; and let n_2, d, r, t, p, and v be the corresponding quantities for the follower, then by the principle of angular velocity ratios we have

$$e = \frac{n_2}{n_1} = \frac{D}{d} = \frac{R}{r} = \frac{T}{t} = \frac{P}{p} = \frac{v}{V} \quad \ldots\ldots\ldots \quad \text{(A)}$$

343. In any case a belt may be used to connect two or more axes of a train, and it may be either open or crossed, a crossed belt being equivalent to two spur wheels in gear, except that the velocity ratio may be affected by the slipping of the belt, and cannot be relied upon.

Now by means of equation (A) we can determine the value of e in any case; for example, let $T_1, T_2, T_3, \ldots T_{m-1}$ represent the numbers of teeth of the drivers on the successive axes, and let $t_2, t_3, t_4, \ldots t_m$ be the numbers of teeth on the corresponding followers. Then we can write, by substituting for $\frac{n_2}{n_1}$, etc., the proper values in terms of T and t.

$$e = \frac{n_m}{n_1} = \frac{v}{V} = \frac{T_1}{t_2} \times \frac{T_2}{t_3} \times \frac{T_3}{t_4} \times \ldots \times \frac{T_{m-1}}{t_m} \quad \ldots \text{(B)}$$

$$= \frac{T_1 \times T_2 \times T_3 \times \ldots \times T_{m-1}}{t_2 \times t_3 \times t_4 \times \ldots \ldots \times t_m}$$

That is, the value of the train is equal to the quotient obtained by dividing the continued product of the numbers of teeth of all the drivers by the continued product of the numbers of teeth of all the followers.

344. It is evident, in trains of this kind (FIG. 1, PLATE XVII), that the number of drivers, as well as the number of followers, is always one less than the number m of the axes. Also, that the arrangement of the factors of the numerator and denominator of the fraction (equation B) expressing the velocity ratio is indifferent, from which it follows that the drivers and followers may be interchanged among themselves without changing the value of the train.

Thus in FIG. 1, PLATE XVII, the value of the train is

$$-e = \frac{200 \times 120 \times 80}{80 \times 60 \times 30} = \frac{40}{3} = 13.\dot{3}33+$$

Similarly, the value of the belt train shown in FIG. 2, PLATE XVII, where the diameters of the pulleys are given in inches, is

$$e = \frac{40 \times 24 \times 16}{16 \times 12 \times 6} = \frac{40}{3} = 13.333+$$

345. GENERAL EXAMPLE. It is not necessary that all the separate velocity ratios be expressed in the same terms as previously explained. For example, let us take a train of six axes, and let:

1°. The first axis revolve once per minute, and the second once in fifteen seconds.

2°. The second axis revolve three times while the third revolves five times.

3°. The third axis carry a wheel of sixty teeth, driving a wheel of twenty-four teeth on the fourth axis.

4°. The fourth axis carry a pulley of twenty-four inches diameter, driving by means of a belt, a pulley twelve inches in diameter on the fifth axis.

5°. The fifth axis have an angular velocity two-thirds of the sixth axis.

$$\text{then } e = \frac{60}{15} \times \frac{5}{3} \times \frac{60}{24} \times \frac{24}{12} \times \frac{3}{2} = 50,$$

or the last axis will make fifty revolutions while the first revolves once.

346. DIRECTIONAL RELATIONS IN TRAINS. This depends on the number and manner of connections of the different axes. When the train consists of spur wheels or pinions on fixed parallel axes, the direction of rotation of the successive axes will be alternately in opposite directions. Thus, if the train has an *odd* number of axes, the first and last will revolve in the same direction; while in a train having an *even* number of axes, the first and the last will revolve in opposite directions.

347. In any arrangement the directional relation can be traced by placing arrows on the different wheels, showing their direction of rotation, and following through the entire combination. It is frequently the case that two separate wheels of a train may revolve about the same axis, and if they revolve in the same direction, they may be connected by means of two other spur wheels revolving on an axis parallel to that of the wheels; while if they revolve in opposite directions, they might be connected by one bevel wheel placed on an axis making an angle with that of the wheels.

348. When an *annular wheel*, that is, a wheel having teeth on the inside of an annular ring, is used in connection with a pinion it will be noticed that they both revolve in the same direction.

349. IDLE WHEEL. A wheel called an idle wheel, which acts both as a driver and follower, is often placed between two others. In such a case the velocity ratio of the two extreme wheels is not affected, but the directional relation is changed, the extreme wheels now rotating in the same direction, while if they geared

directly, they would rotate in opposite directions. In FIG. 3 the two wheels, *A* and *B*, on the shafts *a* and *e* are connected by means of an idle wheel *C*, so that they revolve in the same direction. An idle wheel is often used between two equal wheels placed on parallel axes, which then rotate in the same direction with the same angular velocity. This combination is used in lathes for turning irregular pieces, as spokes for wheels, gunstocks, etc., where a model is made to turn on one axis with exactly the same angular velocity as the piece to be shaped turns on another parallel axis,

350. One of the most frequent applications of trains of wheels is in the screw cutting lathe. Before taking up its description the means of varying the speed at which the parts revolve, demands attention. In the lathe this is accomplished by means of speed pulleys, and what is called the back gear, the nature of which will appear from the description; by its use the different speeds necessary for different classes of work are obtained. See FIG. 4, PLATE XVII.

There is a driving shaft overhead provided with a cone or stepped pulley, and with a fast and loose pulley, which receive the power from the engine, a second cone or stepped pulley *F*, corresponding with the pulley overhead, is fitted on the spindle *CD* of the lathe and rides loose upon it; to this pulley *F* is attached a pinion *G*, which drives a wheel *H;* this wheel *H*, the shaft *AB* and the pinion *K*, constitute the back gear of the lathe, shown here for convenience below in place of in rear of the spindle *CD*. The motion imparted to *H* is communicated by the shaft *AB* through the pinion *K* to the wheel *L* which is fast to the spindle *CD* and turns with it.

The result is that the wheel *L*, and consequently the spindle *CD* revolves much more slowly than the stepped pulley *F*, and that the speed of the spindle *CD* is reduced in the ratio of $\frac{G \times K}{H \times L}$, where *G*, *K*, *H*, and *L* represent the numbers of teeth upon these wheels respectively. Where the lathe is worked at ordinary speeds, the shaft *AB*, and the wheels *H* and *K* are pushed out of gear with *G* and *L*, either by sliding the back gear

AB in the direction of its axis as shown in FIG. 5 or by an eccentric axle *AB* (FIG. 8, PLATE XVII) by means of which a half revolution of *AB* about its own axis, breaks the contact of *H* and *G*, and *K* and *L*, or throws the back gear out of gear. In this case when the back gear is out of gear the pulley *F* is fastened to the wheel *L* (FIG. 6) by an adjustable pin *P* moving in a radial slot *S* in the wheel *L*, so that it can move in an annular groove *G* in the face of the pulley *F*, when the back gear is in gear, and *F* and *L* move independently of each other, or by moving the pin *P* to the other end of its slot, and making it fast, it enters a corresponding socket *x* and *y*, and *F* and *L* turn as one wheel.

✓351. ENGINE-LATHE TRAIN. FIG. 13, PLATE XVII, gives an elevation and end view of the head stock of an engine lathe, showing the method of connecting the spindle or mandrel *m* with the lead screw *L*. The "back gears" *g* and *h*, on the shaft *nn* have been drawn in position above the mandrel, instead of back of it, where they are usually placed, so that one figure will show the whole arrangement, a convention often adopted in drawings of head stocks.

The step pulley *C* turns loose on the mandrel *m*, and carries the gear *f* at its left hand end; the gear wheels *i* and *a* are attached to the mandrel, and turn with it. The gear wheels *g* and *h*, connected by a hollow shaft, turn upon an arbor whose axis is parallel to the axis of the mandrel. This arbor is provided with two eccentric pivots *nn*, so that by turning it slightly, the gears *g* and *h* with their hollow shaft, commonly known as *back gears*, can be thrown in or out of gear with *f* and *i* respectively. An adjustable catch is arranged between the step pulley *C* and the wheel *i*, so that the wheel can be connected directly to the pulley *C* by throwing in the catch, the back gears being out of gear, or indirectly through the back gears, which are now thrown in, the catch being adjusted so that the pulley turns freely on the mandrel. This catch consists of an adjustable pin, moving in a radial slot in the gear wheel *i*, it being held in position by a spring or thumb nut. When the catch is at the outer end of the slot, it works in a circular groove in the pulley *C*; but when it is at the inner end of the slot, it is located in a radial notch cut from the groove, thus compelling *C* and *i* to move together.

Thus, by combining a four-step pulley with back gears, a series of eight speeds can be obtained for the mandrel, four with the back gears in, and four with the stepped pulley only. The stepped pulley and back gear train should be so proportioned that the speeds are in geometric progression.

352. SCREW CUTTING. One of the most frequent uses of an engine lathe is that of cutting screws. To this end, the lathe is provided with an accurately cut screw *L*, called a *lead screw*, which serves to move the carriage of the lathe in a nut *p* made fast to the carriage. This carriage travels upon the bed of the lathe on ways parallel to the axis of the mandrel, and carries the tool-holder; the whole is here represented by the pointer *P*, which illustrates the action just as well.

A train of gear wheels connects the mandrel with the lead screw, an intermediate shaft, as that of *b* and *c*, commonly known as "the stud", being generally used. This stud, *xy*, is then connected to the lead screw by means of the train *cdl*, *d* being an idle wheel, and *c* and *l* change wheels.

Let us suppose the mandrel to make *m* revolutions while the lead screw makes *n* revolutions. *e* being the value of the train connecting the mandrel and the lead screw; then placing the piece *S*, on which the screw is to be cut, on the lathe centres, and attaching it, as shown, to *m*, we have

$$\frac{\text{Pitch of screw on } S}{\text{Pitch of screw on } L} = \frac{n}{m} = e$$

Let the numbers of the teeth in the different wheels be as follows: *a*, 31; *b*, 62; *c*, 84; *d*, the idle wheel; *f*, 56; then

$$\frac{\text{Pitch of } S}{\text{Pitch of } L} = e = \frac{31 \times 84}{62 \times 56} = \frac{3}{4}$$

If we assume the lead screw to have six threads per inch, then

$$\text{Pitch of } S = \frac{3}{4} \times \frac{1}{6} = \frac{1}{8}.$$

As the number of threads per inch is the reciprocal of the pitch, we can write, when *S* represents the number of threads to be cut, and *L* the number on the lead screw per inch,

$$\frac{S}{L}=\frac{m}{n}=\frac{1}{e}\ \ldots\ldots\ldots\ldots\ (A)$$

from which $S=\frac{6\times 4}{3}=8$, as before.

The gears *a* and *b* between the stud and the mandrel, and *S* and *L* being given, we can find *c* and *l* as follows :—

$$\text{From equation (A)}\ \frac{L}{S}=\frac{n}{m}\times\frac{c}{l}=\frac{31}{62}\times\frac{c}{l}=\frac{1}{2}\times\frac{c}{l}$$

$$\text{or}\quad \frac{2L}{S}=\frac{c}{l}$$

Let $L=6$, and $S=24$; then

$$\frac{c}{l}=\frac{12}{24}$$

and we might use wheels for *c* and *l* in the proportions 12 to 24. In this way a table might be calculated for any lathe, and the gears for each pitch arranged in columns as given below :—

Threads per inch.	Gear on Stud.	Gear on Lead Screw.
3	112	28
4	84	28
5	84	35
6	84	42
7	84	49
etc.	etc.	etc.

353. It will be noticed that when the screws *S* and *L* revolve in the same direction, the threads will both be either right or left-handed; while if they revolve in opposite directions, one screw must be right and the other left-handed.

In many lathes it is arranged that either one or two idle wheels can be thrown between *a* and *b* at pleasure, by simply moving an arm placed near *b* in the head stock casting.

354. In gearing the stud with the lead screw, a vibrating slotted arm, FIG. 14, PLATE XVII, is made use of. The wheel *c* is first placed on the stud *x*, and then *l* is placed on the lead screw at *t;* a wheel *d* is then selected from among the change wheels, and placed on the movable stud *z*, of the arm *w*, the stud being adjusted so that *c* and *d* gear with each other; the arm is then

swung over, until *d* and *l* gear with each other, and clamped in position by means of the screw *s*.

355. CLOCKWORK. A familiar example of the employment of wheels in trains is seen in clockwork. FIG. 1, PLATE XVIII, represents the train of a common clock; the numbers on the different wheels denote the number of teeth in the wheels on which they are placed.

The verge, or anchor *o*, vibrates with the pendulum *p*, and if we suppose the pendulum to vibrate once per second, it will let one tooth of the escape wheel pass for every double vibration, or every two seconds. Thus the shaft *a* will revolve once per minute, and is suited to carry the second hand *S*.

The value of the train between the axes *a* and *c* is $e = \frac{8 \times 8}{60 \times 64} = \frac{1}{60}$ or the shaft *c* revolves once for sixty revolutions of *a;* it is therefore suited to carry the minute hand *M*. The hour hand *H* is also placed on this shaft *c*, but is attached to the loose wheel 64 by means of a hollow hub. This wheel is connected to the shaft *c* by means of a train and intermediate shaft *e*.

The value of this train is $e = \frac{28 \times 8}{42 \times 64} = \frac{1}{12}$

That is, the wheel 64 makes one-twelfth of a revolution for one revolution of the shaft *c;* it is therefore suited to carry the hour hand *H*. Lastly, the drum *d*, on which the weight cord is wound, makes one revolution for every twelve of the minute hand *M*, and thus revolves twice each day. Then, if the clock is to run eight days, the drum must be large enough for sixteen coils of the cord. The drum is connected to wheel 96 by means of a ratchet and click, so that the cord can be wound upon the drum without turning the wheel.

Clock trains are usually arranged as shown in the figure, the wheels being placed on shafts, usually called "arbors", whose bearings are arranged in two parallel plates, shown in section, which are kept the proper distance apart by shouldered pillars (not shown) placed at the corners of the plates. When the arbor *e* is placed outside, as shown, a separate bearing is provided for its outer end.

356. Trains of wheels are often used in cranes for raising heavy weights. In FIG. 2, PLATE XVIII, let r equal the distance from the axis of the drum to the centre line of the hoisting rope w, l the length of the handle arm, both being expressed in inches, and $p = 30$ pounds, the force exerted at the handle; then neglecting friction and applying the principle of work, we have, calling the pull of the rope w,

$$w \times r = p l e = 30 \times l \times \frac{40}{3}$$

$$\text{as } e = \frac{200}{80} \times \frac{120}{60} \times \frac{80}{30} = \frac{40}{3}$$

$$\therefore \cdot w = \frac{30 \times l}{r} \times \frac{40}{3}$$

When $r = 6''$, and $l = 18''$, $w = 1200$ pounds.

357. FREQUENCY OF CONTACT BETWEEN TEETH. HUNTING COG. Let T and t be the numbers of the teeth on two wheels in gear, and let $\frac{T}{t} = \frac{a}{b}$ when reduced to its lowest terms. It is evident that the same teeth will be in contact after a revolutions of t, and b revolutions of T. Therefore the smaller the numbers a and b, which express the velocity ratio of the two axes, the more frequently will the same pair of teeth be in contact.

Assume the velocity ratio of two axes to be nearly as 5 to 2. Now if we make $T = 80$ and $t = 32$, we will have

$$\frac{T}{t} = \frac{80}{32} = \frac{5}{2} \text{ exactly,}$$

and the same pair of teeth will be in contact after five revolutions of t, or two revolutions of T.

If we now change T to 81, then $\frac{T}{t} = \frac{81}{32} = \frac{5}{2}$ very nearly, the angular velocity being scarcely distinguishable from that which it was originally. But now the same teeth will be in contact only after 81 revolutions of t, or 32 revolutions of T.

The insertion of a tooth in this manner was an old contrivance of millwrights to prevent contact between the same pair of teeth too often, and was supposed to insure greater regularity in the wear of the wheels. The tooth inserted was called a *hunting cog*, because a pair of teeth, after being once in contact, would gradually separate and then approach by one tooth in each revolution, and thus appear to hunt each each other as they went round.

In clock trains, where the velocity ratio must be exact, the above principle can be employed only to a limited extent.

358. METHOD OF DESIGNING TRAINS. This is the reverse of the problem of finding e, the value of the train, as here the value e is given and the number of wheels and axes is to be fixed. Then we have for a train of wheels in external contact

$$e = \frac{T_1 \times T_2 \times T_3 \times \ldots \times T_{m-1}}{t_2 \times t_3 \times t_4 \times \ldots \times t_m}$$

LEAST NUMBER OF AXES. The number of axes will depend upon the limits between which the numbers of the teeth are allowed to vary. For example, let T be the *greatest* number of teeth that can be given to a wheel, and t the *least* that can be given to a pinion, and suppose the wheels to act as drivers. The number of axes being m, we will have $m-1$ wheels and $m-1$ pinions. Hence.

$$e = \frac{T \times T \times T \text{ to } (m-1) \text{ factors}}{t \times t \times t \times \text{to } (m-1) \text{ factors}} = \left(\frac{T}{t}\right)^{m-1}$$

whence $\quad \log e = (m-1)(\log T - \log t).$

$$\therefore \quad m = 1 + \frac{\log e}{\log T - \log t} = 1 + \frac{\log e}{\log \frac{T}{t}}$$

The least number of axes, under the assigned conditions, is the value of m thus found, if this value is a whole number; or the next larger whole number, if m is fractional. The values of T and t cannot be fixed by any general rule, as they are governed by considerations which vary with the nature of the machine. It also often happens that the fraction giving the velocity ratio will not admit of being divided into factors so nearly equal as to allow the use of the limiting number of axes.

359. The previous discussions outline the method of designing a crane for lifting weights. Having given a train of wheels whose value is 360, the smallest wheel to have 12, the largest 108 teeth, to design a crane for lifting weights, and to ascertain the relation

between the power and resistance; the power supposed to be 60 pounds, is applied with an effective radius of 24 inches, and the radius of the drum, on which is wound the cord that supports the weight, is 6 inches.

The equation of work gives

$$2\pi P \times R \times e = 2\pi W \times r$$ ✓

P = power, R = length of lever arm of the power,
e = value of the train, W = weight,
r = radius of drum.

$$\therefore \quad W = 60 \times 2 \times 360 \times 2 = 86400 \text{ pounds.}$$

Applying the formula for the number of axes

$$m = 1 + \frac{\log e}{\log \frac{T}{t}} = 1 + \frac{\log 360}{\log 9} = 3 +$$

The number of axes must be 4, and there must be 3 wheels and 3 pinions; factoring the value of the given train, $e = 360$ we have

$$e = 360 = \frac{5 \times 8 \times 9}{1 \times 1 \times 1}$$

multiplying numerator and denominator by $12 \times 17 \times 13$, and arranging so as to have no common factors possible in pairs, so as to avoid frequency of contact, we have

$$e = 360 = \frac{85}{12} \times \frac{104}{17} \times \frac{108}{13}$$

which gives us the number of teeth on the wheels and pinions. The first axis carries the pinion 12 and is the point of application of the power, by means of the crank arm with an effective radius of 24 inches. The second axis carries the wheel 85 gearing with the pinion 12, and also carries the pinion 17 which gears with the wheel 104 on the third axis which also carries the pinion 13, which gears with the wheel 108 on the fourth axis, which axis carries the drum, with an effective radius of 6 inches, on which is coiled the rope that raises the weight. The arrangement of the train is shown in FIGS. 3 and 4, PLATE XVIII.

360. Again, to design a crane that shall lift 20 tons, 44800 lbs. In the equation of work

$$2\pi PRe = 2\pi Wr$$

let us assume $P = 60$ lbs., $R = 2$ feet, and $r = 6$ inches, these being values that can be given in practice to these quantities. This enables us to determine the value of a train that will give the desired power to lift 20 tons.

$$e = 44800 \times \tfrac{1}{2} \div 60 \times 2$$

$e = 186$ to within less than unity. Assume the limits of the number of teeth to be 12 and 108.

$$\text{Then} \quad m = 1 + \frac{\log 186}{\log \frac{108}{12}} = 3+$$

Therefore we have 4 axes, 3 wheels and 3 pinions; factoring the value of the train $e = 186 +$, we have

$$e = 186 = \frac{6}{1} \times \frac{5}{1} \times \frac{6.2}{1}$$

Taking as multipliers the numbers 17, 13 and 15, we have after arranging as in previous case

$$e = \frac{102}{15} \times \frac{65}{17} \times \frac{93}{13} = 186$$

Our pinions will have 13, 17 and 15 teeth respectively and the wheels will have 93, 65 and 102 teeth respectively, and the mechanical arrangement will be similar to the previous case.

In determining the number af axes, we can more conveniently obtain it by dividing the value of e, expressed decimally, successively by $\frac{T}{t}$ until a quotient less than unity is obtained, and then adding one to the number of divisions necessary, and the final result is the number of axes required.

In the case just described $e = 186$; $\frac{T}{t} = 9$.

$$186 \div 9 = 20.66,$$
$$20.66 \div 9 = 2.29,$$
$$2.29 \div 9 = .25.$$

3 divisions were necessary,

$$\therefore \quad m = 3 + 1 = 4.$$

The factoring, selection of multipliers, and arrangement of factors, may be varied according to the conditions imposed, or as the judgment of the designer may indicate.

In light work where frequency of contact is admissible, not so much care is necessary in avoiding common factors in the elementary combinations.

PLATE XIX.

Aggregate Combinations.

361. Aggregate combination is a term used to denote those assemblages of pieces in mechanism, in which the motion of one follower is the resultant of motions impressed upon it by more than one driver. The number of independently acting drivers which impress directly a compound motion on one follower cannot be greater than three; because each driver determines the circumstances of motion of at least one point in the follower, and the determination of the circumstances of motion of three points in a body determines the motion of the whole body. In most cases which occur in practice, the number of independent drivers which act directly on one follower is two.

362. Let us recall some definitions bearing on this subject. The frame of a machine is a structure which supports the moving pieces, and regulates the path or kind of motion of most of them directly. The moving pieces may be distinguished as *primary* and *secondary* pieces; primary pieces are those which are directly carried by the frame, and have their motion wholly guided by their connection with the frame; secondary pieces are those which are carried by other moving pieces, or have their motion not wholly guided by their connection with the frame. For example, the crank shaft and the piston rod of a steam engine are primary pieces; the connecting rod is a secondary piece.

363. Any follower which has a compound motion directly communicated to it by more than one primary piece must be a secondary piece, and its motion at any instant is the resultant of the motions separately impressed upon it by the pieces which act as drivers.

364. Very often an aggregate combination is of the nature of a train and although a secondary piece receives in the first instance a compound motion from 2 or 3 primary pieces yet that secondary piece communicates motion in the end to a primary piece.

In such cases the motion of that last primary follower may be determined by finding the motions which would be communicated to it through the intermediate secondary piece or pieces by the several primary drivers acting separately and taking the resultant of these motions. A secondary piece in an aggregate combination has very often a form like that of a primary piece and is only distinguished from a primary piece by the fact that its bearings, instead of being carried by the fixed frame, are carried by a moving frame; that moving frame being one of the primary pieces from which the secondary receives its motion. For example, a wheel may turn about an axis, which axis is carried by an arm that turns about another axis, as in FIG. 6, PLATE XX.

365. When such a secondary piece is to drive or to be driven by a primary piece, or another secondary piece not carried by the same moving frame, special contrivances, which may be called *shifting trains*, have to be used in order to keep up the connection between the two pieces during their various changes of relative positions. For example:

1. When two pieces turning about parallel axes are connected by toothed gearing, and one of them is free to shift its position along its axis relatively to the other, the long or broad pinion may be used. In (FIG. 1, PLATE XIX) *AA* and *BB* are a pair of parallel axes, *C* is a spur wheel on *AA* and *D* is a broad pinion on *BB*, and the breadth of the pitch surface of *D* is made greater than that of *C* by a length equal to the distance through which *D* is capable of being shifted longitudinally.

2. When a toothed wheel *CC* (FIG. 2, PLATE XIX) gears with a rack *DD* and either the rack is to be capable of turning about an axis *BB* parallel to its pitch line, or the axis *A* of the toothed wheel *CC* is to be capable of being revolved about the axis *BB* by means of an arm *FA*, the circular rack is to be used, being as represented in the figure, a solid of revolution generated by the rotation of the trace of the rack teeth about *BB* as an axis. The pitch line *DD* becomes the trace of an imaginary pitch cylinder generated by its revolution about *BB* as an axis, and the pitch point *E* is the point of contact of that cylinder with the pitch cylinder of the wheel *CC*.

366. By fixing a broad pinion on one part of a shaft and a circular rack on another, that shaft may receive at the same instant two independent motions: one of rotation about its axis, and one of translation along that axis, from two different spur wheels; the resultant motion being a helical motion; this is one of the simplest aggregate motions.

367. Every case of curvilinear motion of a point is of a compound character, resulting from the superposition of two or more rectilinear motions. It often happens in machinery that some revolving wheel or moving piece becomes the recipient of more than one independent motion, and that such different movements are concentrated upon it at the same instant of time. This motion is of a compound or aggregate character.

368. As an example let us consider the mechanical contrivance called the *Lazy Tongs*, (FIG. 3, PLATE XIX), the rapid advance of the ends *A* and *B*, when the handles *C* and *D* are made to approach each other is due to to the fact that these points *A* and *B* are the recipients of the sum of the longitudinal components of the circular motion which takes place at the points *aa'*, *bb'*, *cc'*, when the handles *C* and *D* are made to approach each other. When *C* and *D* approach, the points *a* and *a'* describe arcs of circles about *x* as a centre. This circular motion can be resolved into two components, one in the direction of *CA*, which is carried throughout the combination, being augmented by a similar component at the points *bb'* and *cc'*, so that *A* and *B* receive the aggregate of all these separate motions.

369. A wheel rolling upon a plane is a case of aggregate motion, the centre of the wheel moves parallel to the plane, the wheel itself revolves about its centre and the aggregate result of these two motions is rolling.

370. The instantaneous axis of a body rolling on a plane is the point of contact of the rolling body and the plane, hence a point on top of the rolling body is moving with twice the linear velocity of the centre. If then a timber be moved longitudinally upon rollers, the travel of the beam will be twice as great as that of the rollers. This principle is employed in some printing machines, where the table receives its reciprocating motion by means of a crank and connecting rod. The path of the table is

doubled by employing the principle just enunciated. Here (FIG. 4, PLATE XIX) a toothed wheel Q is attached to the end of the connecting rod PQ, so that it can turn freely on its centre Q. The wheel is made to revolve between two racks A and B. The rack A is a primary piece and is fixed to the framework of the machine, while the secondary rack B is movable and carries the table. The rack B then moves through exactly twice the space that it would describe if connected with the point Q. The point of contact of the wheel Q with A is the instantaneous axis of the point Q, hence any point in B as the point of contact of Q and B, and hence the entire rack B, being twice as far from A as Q, will move twice as far as Q. (Only part of the teeth on Q are shown).

371. The usual classification of aggregate combinations is according to their purpose.

They may effect aggregate velocities or aggregate paths.

1. When an aggregate velocity is the object aimed at, the final piece of the train is usually a primary piece, whose relative velocity, by means of an aggregate combination, is made either to have a constant value or to vary according to law, which it might be difficult or impossible to realize by means of a train of elementary combinations only.

2. When an aggregate path is the object aimed at, a point in a secondary piece is made by means of an aggregate combination to move in a path which may be different from that which a point in a primary piece would describe.

The only paths which points in primary pieces can describe are straight lines, circles, and helical paths, and other curved paths must be described by the help of aggregate combinations.

Sometimes it is found convenient to use aggregate combinations for describing exactly or approximately the elementary paths, the straight line, circle, or helix. As an example, the aggregate combinations called parallel motions are used to constrain a point to move exactly or sensibly in a straight line.

372. Two cams might be made, by means of a system of levers and rods to act on the same point in a secondary piece, the motions governed by each cam being in lines situated in one plane and making right angles with each other; the aggregate or

resultant motion of the point so governed would be of a nature to trace any plane curve within the limits of its movements.

373. A combination in aggregate *linkwork* is usually of the following kind: a bar or other rigid body capable of moving parallel to a given plane, has two of its points connected by means of rods with two drivers; a third point is connected by means of a rod with a follower. The motions of the first two points, as compared with those of their drivers, are determined by the principles of elementary combinations in linkwork, and so also is the motion of the follower, as compared with that of the third point; but the determination of the motion of the third point from that of the first two points must be found by reference to the instantaneous axis or some equivalent process.

374. In most of the particular cases of the aggregate velocities obtained by linkwork which occur in practice, the three points in the bar are either situated in one straight line, to which their motions are perpendicular, or are so nearly in a straight line that their comparative motions, as determined on the supposition that they are so exactly, are sufficiently near the truth for practical purposes.

375. Let A, B, (Fig. 5, Plate XIX) be the two points, whose velocities at a given instant are given, and let C be the third point. Draw Aa and Bb perpendicular to AB and of lengths proportional to the given velocities, and in the given direction; from ab draw Cc perpendicular to AB and cutting ab in C; Cc will then represent the velocity of C. To determine analytically the velocity of C, consider the motions to take place separately, and let a, b and c represent the velocities of A, B and C, respectively. Then considering B as fixed and A to move, C will have a velocity that will be to the velocity of A as BC to AB

$$\therefore\ c = a\,\frac{BC}{AB}$$

If we consider A as fixed and B to move, C will have a velocity $= b\,\frac{AC}{AB}$, considering motion in one direction as positive and in the opposite direction as negative, the resultant velocity of C would be the algebraic sum of the two component velocities, or the two simple velocities due to the movements that give the aggregate motion, or

$$c = \frac{aBC + bAC}{AB}$$

That is, the velocity of C may be regarded as the resultant of two component velocities, $a\frac{BC}{AB}$, which would be its velocity if B were fixed, and $b\frac{AC}{AB}$, which would be its velocity if A were fixed.

376. TRAINS OF AGGREGATE LINKWORK may be used to combine any number of component motions. For example, in FIG. 6, PLATE XIX, A, B, D, F, receive motion from four different drivers; C has a motion whose components depend on the motions of A and B, and E has a motion whose components depend on the motions of D and F, and G has a motion whose components depend on the motions of C and E, and therefore on the motions of A, B, D and F jointly.

The following are well known examples of aggregate combinations.

377. A COMPOUND SCREW consists of two screws cut upon the same spindle and each having a nut fitted upon it.

The screw turns, one of the nuts is usually fixed, so that the screw in turning in that nut is made to advance; the other nut slides, but does not turn; and the sliding motion of the second nut relatively to the first nut, is the resultant of the advance of the screw relatively to the first nut, and of a motion equal and opposite to the advance of the screw relatively to the second nut; that is to say the second nut moves relatively to the first nut as if it were acted upon by a single screw of a pitch equal to the difference between the pitches of the two given screw threads, supposing these threads are both right, or both left-handed. But if the threads are contrary handed, *i. e.*, one right and one left-handed or *vice versa*, the resultant motion of the second nut is equal to the motion it would have if it were acted upon by a single screw of a pitch equal to the sum of the pitches of the two screw threads. Where the threads are both right-handed or left-handed, the compound screw is called a *differential screw*. A right and left-handed screw is often seen in combination for the purpose of bringing two pieces together.

In FIG. 7, PLATE XIX, the swinging of the arm AB by means of the contrary handed screws that are moved by it, causes the nuts E and F to approach or recede, according to the direction in which AB is revolved.

378. Any system of *pulleys* will form an example of aggregate motion. Take the single movable pulley in FIG. 8, PLATE XIX, when W is raised one inch, the block is raised one inch, but the end of the line P is lifted 2 inches, due to the aggregate effect of the raising of the block or weight and the turning of the pulley on its axis, and the consequent transfer of line.

379. Another example is the *Chinese Windlass*, FIG. 9, PLATE XIX. Here a rope is coiled in opposite directions around two axles A and B of unequal size ; the rope is consequently unwound from one axle while it is being wound up by the other, and the weight may be made to rise as slowly as we please. Let R and r be the radii of the two axles, then the weight W moves through $\pi(R-r)$ for each revolution of the axle.

The practical objection to this windlass consists in the great length of rope required during the operation. In the ordinary windlass the amount of rope coiled upon the barrel, represents the height through which the weight is raised, whereas here we begin by winding as many coils on the smaller barrel as the number of turns we intend to make with the crank handles, and then at the close of every turn a length of rope is coiled on the large barrel equal to $2\pi R$, but the weight has only been lifted through $2\pi(R-r)$.

If $R = 11$, and $r = 10$, then the amount of rope wound on to the larger barrel in any number of turns bears the same proportion to the space through which the weight is lifted that $2\pi R$ does to $\pi(R-r)$ or that 22 does to 1.

380. The employment of *differential pulleys* is to avoid the employment of so much rope in making small lifts. In the Chinese windlass, just described, one end of the rope is fastened to the axle A and the other end to the axle B. Had these two ends been brought together the supply of rope necessary for B, might

have been drawn from *A*, and the expenditure much decreased thereby.

381. A wheel rolling on a plane is a case of aggregate motion, the centre of the wheel moving parallel to the plane, and the wheel itself revolving upon its centre. The resultant of these two motions gives the aggregate result of rolling.

Fig. 10, Plate XIX shows a contrivance devised with a view of obtaining great speed in cars propelled by ropes, at the same time requiring but a comparatively slow velocity of the propelling rope.

Two grooved wheels *A* and *B*, of different diameters, are centred on the same axis *c*, and are arranged so that they can be made to turn separately or together ; the whole is attached to the body of the car by means of the shaft *c*. A rope passes from the pulley *C*, located at one end of the route, around the wheel *B*, then around the pulley *D*, and back around the wheel *A* to the pulley *C*. Now if we suppose the pulley *D* to turn right-handed, the rope will move as indicated by the arrows ; and if the two wheels are made fast to each other, the point *a* will tend to move to a', and the point *b* to b', $aa' = bb'$, giving a resultant motion cc' to the axis of the wheels. In this case the instantaneous axis is midway between *a* and *b*.

Let the diameters of the wheels *A* and *B* be as 7 to 6, and let *v* be the velocity of the rope, and *c* the velocity of the shaft *c*, then *c*, being outside of *ab*, we have for the velocity of the axis of the wheels

$$c = \frac{v \times ac + v \times bc}{ab} = 2v \left(\frac{6}{2} + \frac{7}{2}\right) = 13v.$$

By the principle of the instantaneous axis,

$$v : c = \frac{ab}{2} : ac + \frac{ab}{2}$$

$$c = v\frac{13}{4} \times 4 = 13v$$

Thus when one yard of rope is drawn over the pulley *D*, the car will be drawn along 13 yards. To stop the car it will only be necessary to disconnect the wheels *A* and *B*.

382. WESTON'S DIFFERENTIAL PULLEY BLOCK. Here the principle of aggregate motion is made use of for lifting heavy weights with a small expenditure of force.

It consists of a double geared chain pulley AB (FIG. 11, PLATE XIX) with a fixed axis a and a movable grooved pulley or sheave C, from the axis of which the weight to be lifted is suspended. The two pulleys are connected by an endless chain, as shown. The diameter of the chain groove A is less than that of B, and the diameter of C is a mean between that of A and B. The pitch lines only of the pulleys and chain are shown in the figure.

383. VELOCITY RATIO. Let v represent the velocity of the chain in the direction of the arrow; then if we consider the point c as fixed, hauling down upon the chain at e would raise the pulley C with a velocity $= \frac{v}{2}$. But c being rigidly connected with e, it moves downward with a velocity $= v \times \frac{ca}{ea}$. Then considering f as fixed, this would give to C a downward velocity $= -\frac{v}{2}\,\frac{ca}{ea}$, upward motion being considered positive, and downward negative. The resultant velocity of C is

$$b = \frac{v}{2} - \frac{v}{2}\,\frac{ca}{ea} = v\,\frac{ea - ca}{2ea}$$

and the velocity ratio $= \frac{b}{v} = \frac{ea - ca}{2ea}$

Let the diameter of the pitch circle of $A = 14''$, and that of $B = 15''$, and let $P = 100$ lbs. be the pull on the chain e, and W the weight that can be raised, neglecting friction. Then by the principle of work we have

$$Pv = Wb \;\therefore\; W = \frac{Pv}{b}$$

$$\text{or} \quad W = P\,\frac{2ea}{ea - ca} = 100\,\frac{15}{7.5 - 7} = 3000.$$

Thus a pull of 100 lbs. on the chain will raise a weight of 3000 lbs., friction being neglected. The motion of C might have been derived as follows: Suppose the wheels A, B to make one revolution left handed, then an amount of chain $= \pi ef$ will be taken up at f and an amount $= \pi cd$ will be delivered at c, thus giving C a motion

$$b = \frac{\pi}{2}(ef - cd)$$

for a motion of the chain at $e = v = \pi ef$, and thus the velocity ratio is

$$\frac{b}{v} = \frac{ef - cd}{2ef} = \frac{ea - ca}{2ea} \text{ as above.}$$

One advantage of this block is that it will retain the load, the pull at f, having but little more leverage than the equal pull at c in the block as constructed, is not sufficient to more than overcome the friction of the chain and the pin a.

384. The following mechanism for drilling is an application of the principle of aggregate motion.

In FIG. 12, PLATE XIX, AD is the drill spindle which is driven by the bevel wheel B, E and F are two worm wheels embracing the screwed portion of the spindle AD on both sides. These worm wheels are hollowed out on their perimeters so as to fit the small thread on AD, being cut so that the cross section of their teeth is the same as that of the screw thread on AD.

Now if E and F be prevented from turning, they will form a nut through which the drill spindle AD will screw itself rapidly. If on the other hand E and F be allowed to turn freely, the drill spindle AD will set them in motion, and the nut will be virtually eliminated.

The drill spindle is the recipient of two equal and opposite motions; it is depressed by screwing through the nut formed by E and F, and it is elevated by the rotation of the wheels E and F.

If the rotation of the wheels be in any degree checked by the application of friction, equality between the upward and downward components is destroyed, and the aggregate effect is that the drill spindle descends to a corresponding extent. A friction brake regulated by a screw restrains the motion of E and F and gives perfect command over the motion of the spindle.

When B is at rest, rotation of the worm wheels act as a rack and pinion, and the drill can be rapidly lowered to its work.

385. In drilling tools the spindle should rotate rapidly but advance slowly. The principles of aggregate combinations accomplish this most readily.

Conceive a nut *N*, FIG. 13, PLATE XIX, to be placed upon a screw bolt *FG*, and to be so held in a ring or collar that it can rotate freely without being capable of any other motion.

If the nut be fixed, and *FG* be tuued in the direction of its screw thread, the bolt *FG* must advance through the nut. If the screw be prevented from turning, and the nut be made to rotate in the same direction, the bolt will take a motion in a direction the reverse of its former motion. If by any mechanical combination, different amounts of rotation be impressed simultaneously on the bolt and the nut, the bolt will receive the two longitudinal movements simultaneously, and the aggregate motion will be the algebraic sum of the longitudinal movements due to the rotation of the bolt and the nut respectively.

386. In FIG. 14, PLATE XIX, suppose the wheels *D* and *C* to be attached to the bolt and the nut respectively, *i. e.*, *D* is fast to *FG* and *C* is fast to *N*. Suppose *D* and *C* are driven by the broad pinion *A* and the gear wheel *B* respectively, which are fixed on the same axis *SS*, and let *A*, *B*, *C* and *D* represent the numbers of teeth on the respective wheels. Then if a be the number of rotations of the shaft *SS* with the pinions *A* and *B*, while the nut makes m rotations and the wheel *D* and the bolt makes n rotations, we shall have

$$\frac{m}{a}=\frac{B}{C} \text{ and } \frac{n}{a}=\frac{A}{D}$$

Therefore a rotation of *SS* will cause a longitudinal movement of the bolt *FG* through a space equal to $(m-n)\times$ the pitch of the screw, equal to $a\left(\frac{B}{C}-\frac{A}{D}\right)\times$ the pitch of the screw, obtained by substituting for m and n their values.

387. Another drilling machine embodying the principles of aggregate motion is shown in FIG. 15, PLATE XIX.

The drill spindle *S* has a screw thread cut upon it. A groove is cut longitudinally along the spindle *S*, and a corresponding projection or key, on the interior of the boss of the wheel *D*, fits into this groove. Thus the spindle *S* can traverse the wheel *D*, although the spindle *S* and the wheel *D* must turn together. A nut *H*, in the form of a pipe having a wheel *C* fastened to it, and turning with it, receives the spindle *S*, see FIG. 16, PLATE XIX. If *C* is stationary, and a pinion *A*, turning as indicated, engage the wheel *D*, it will cause the spindle to move in the direction of its axis downwards towards the work.

If a second pinion, turning in the same direction as *A*, act upon *C*, and *D* remain stationary, it will cause the nut or pipe *H* to rotate instead of the spindle *S*, and the spindle being unable to turn with it on account of its being keyed to *D*, will rise as long as it is prevented from rotating. The aggregate motion due to the combination of these two motions, properly adjusted, will cause the drill spindle to descend slowly accompanied by a rapid rotation, which is required in the process of drilling.

388. FIG. 17, PLATE XIX, shows the manner of applying this principle. The wheels *A* and *B* are moved together; the wheel *A* tends to lower the spindle *S* and to rotate it, the wheel *B* tends to raise the spindle, and as *A* is greater than *B*, the spindle descends with an aggregate motion due to the difference of the motions of translation imparted by *A* and *B*, and rotates with the motion of rotation received from *A*.

The motions of *A* and *B* are obtained from the driving pulleys *N* and *L*. *I* is an idle pulley. *N* and *A* are keyed to the shaft *PP*. *L* and *B* may be one casting and loose on the shaft *PP*. Therefore when the belt is on *N*, the shaft *PP* revolves, the wheel *A* also turns, turning *D*, and the drill descends with a rapid motion of translation and rotation. When the belt is on *L*, the wheels *L* and *B* turn, causing *C* to rotate, and the spindle *S* ascends with no motion of rotation. When the belt is partly on *N* and partly on *L*, the spindle takes the aggregate motion of slow translation downwards, and the rapid rotation, and the drilling proceeds.

The feed could be varied by means of stepped pulleys.

In the drill as described the rate of feed is invariable.

PLATE XX.

Epicyclic Trains.

389 When rotation is to be transmitted from a fixed axis to a shifting axis or from one shifting axis to another, and the relative motion of the two axes is such that their distance apart and the angle which their directions make with each other do not change, or, in other words, when one of the two axes revolves about the other as if it were carried by a rigid rotating arm, whose axis of rotation is the other axis, the connection between those axes may be kept up by means of one rigid frame which carries any combination or train combination or train of mechanism suitable for transmitting rotation from one axis to the other, such a frame is called a *train arm*, and such a combination of moving pieces and a train arm is called *an epicyclic train.*

390. The simplest case is that in which the train carried by the train arm is such that the two axes connected by it are parallel and the angular velocities of the pieces which turn about these axes are equal and in the same direction.

391. In FIG. 1, PLATE XX, the plane of projection is supposed to be normal to the two axes to be connected, and A and B are the traces of those two axes, and AB is their common perpendicular. A movable frame or train arm connects the bearings of the axes with each other, so that the distance AB is invariable; and that frame carries a train of mechanism, xyz, such as to transmit the angular velocity of the piece that turns about A unchanged in amount and direction to the piece which turns about B. For example those pieces may have pairs of parallel and equal cranks linked together by coupling rods; or they may be equal and similar pulleys connected by an open belt; or equal and similar gear wheels with an intermediate or idle wheel gearing with both and carried by the train arm. See FIG. 2, PLATE XX.

The result is that while the train arm turns into any other position, as Ab, FIG. 1, the angular velocities of the pieces which rotate on the axes *A* and *B* respectively, continue to be equal in magnitude and identical in direction.

392. An epicyclic train for producing a uniform aggregate velocity ratio may consist essentially of four parts, whose general arrangement may be held to be represented by the diagram in FIG. 3, PLATE XX, viz: The primary wheels *B* and *C* turning on the same axis *O*, but with different uniform velocities; the train arm *A* being a movable frame turning with a uniform velocity about the same axis *O*; and the shifting train of secondary pieces, carried by the train arm *A*, and transmitting a uniform velocity ratio from *B* to *C* in the manner of an ordinary train, as *xyz* FIG. 1.

393. FIG. 4, PLATE XX. This shifting train may consist of ordinary mechanisms, such as circular gear wheels, whether spur, bevel or skew bevel, skew gearing, pulleys and belts, or links with equal parallel cranks and coupling rods. So that any desired velocity differing in magnitude and direction can be transferred from *B* to *C* by means of the train arm *A*, and shifting train, *xy*.

394. EPICYCLIC TRAINS may be defined as trains of wheels in which some or all of the wheels have a motion compounded of a rotation about an axis and a revolution, or translation, of that axis.

The wheels are usually connected by a rigid link, such as *D* (FIG. 5, PLATE XX). This link is the ***train arm***, and it often rotates upon the axis of the first wheel, *A*, of the train; the last wheel of the train may or may not be placed upon this axis.

In what follows we will consider a wheel to have made one turn or rotation when an arrow placed upon it, as in FIG. 6, PLATE XX, comes again into a position parallel to its first position with the head upward, after a continuous angular motion, either right oı left-handed. The word ***turn*** will be used in place of ***revolution***, as it is much shorter; right-handed turns will be considered ***positive***, the left-handed turns ***negative***.

Epicyclic trains are easily understood if the resultant epicyclic motion is resolved into its components and then each is discussed separately, one component motion taking place after the other. Before deducing a general formula for these trains, some examples will be discussed by the above method.

395. TWO-WHEEL TRAIN. Such a train is shown in FIG. 6, PLATE XX. Let the wheel A be fixed, and suppose the arm D to turn upon the axis of A and to make one turn right-handed; required the number of turns of B, and their direction. To make the problem general, let the numbers of the teeth on A and B be represented by T and t; then the value of the train is $e = \frac{T}{t}$

1. Suppose the combination locked so that the wheels cannot turn in relation to each other or to the arm, and turn the whole once *R.H.* (right-handed); then the first line of table (A) represents the turns of each of the three parts of the train A, B, and D.

2. Now we have done with D what was desired, but in so doing the wheel A has been turned once *R.H.*, and it should have remained still. Therefore, to put A where it should be, unlock the wheels and turn A, once *L.H.*, $= -1$, keeping D still; then B will make $\frac{T}{t}$ turns *R.H.* The result of the second motion is shown in the second line of table (A).

	A	B	D	
1. Train locked	$+1$	$+1$	$+1$	
2. Arm fixed	-1	$+\frac{T}{t}$	0	Table (A)
3.	0	$+1+\frac{T}{t}$	$+1$	

3. Finally, by adding the different columns, we obtain line 3, from which we see that B makes $1 + \frac{T}{t}$ turns *R.H.* for one *R.H.* turn of the arm D, the wheel A being fixed.

If we assume the wheels to be of the same size and to have the number of teeth represented in the figure, we will have, under the same conditions as above, the following table; as $T = t$,

	A	*B*	*D*
Train locked	+1	+1	+1
Arm fixed	−1	+1	0
	0	+2	+1

Thus, in this case, *B* will make +2 turns or 2 turns *R.H.* Tables like the above will be used in what follows.

396. FIG. 7, PLATE XX, shows an application of the two-wheel train commonly known as the *Sun and Planet Wheels*, first devised by Watt to avoid the use of ~~a crank; in his~~ device the pin *b* worked in a circular groove *gg* around the centre *a*, which took the place of the link *ab*, and kept the two wheels in gear. The rod *B* is attached to the beam *XY* of the engine, and *a* represents the engine shaft. While in this case we cannot say that the wheel *C* does not turn, yet its action on the wheel *D*, for an interval of one rotation of the arm, is the same as though it did not turn, as the position at the start and stop is the same. Then to find the turns of *D*, for one turn of the link *ab R.H.*, we will first disconnect the rod and lock the train, thus obtaining the first line of the following table. Then as *C* has been turned +1, we unlock the train, fix the arm, and turn *C*, −1, giving the second line.

	C	*D*	*ab*
Train locked	+1	+1	+1
Arm fixed	−1	+1	0
By addition	0	+2	+1

we find the wheel *D* makes 2 turns *R.H.*

397. THREE-WHEEL TRAIN. A three-wheel train, formed from the two-wheel train by inserting the idle wheel *B*, is shown in FIG. 5, PLATE XX. In this case let us assume that the arm makes +10 turns and the wheel *A* is fixed; required the turns of *B* and *C*. Proceeding as before, we have

	A	*B*	*C*	*D*
Train locked	+10	+10	+10	+10
Arm fixed	−10	+10	−10	0
	0	+20	0	+10

giving +20 turns for *B*, 0 turns for *C*. Then in this case, when the value of the train $e = 1$, the wheel *C* does not turn, but has a motion of translation about the axis of *A*.

This train is made use of in rope-making machinery, to give the bobbins which carry the strands such a motion that the strands are not untwisted in laying the rope, the twist in laying being opposite to that of the strands; also in wire-rope machinery, where the individual wires cannot be twisted in the laying. The arrangement usually adopted is shown in FIG. 8, PLATE XX. The bobbins for the strands, or wires (shown dotted), are attached to the wheels *B* by means of the spindles *b*, turning in bearings in the large disc *D*, which turns with the rope-laying block on the shaft *a*. The idle wheels *C*, turning on pins placed in the disc, connect the wheels *B* with the stationary wheel *A*, on the axis *a*, having the same number of teeth as the wheel *B*. Then, on rotating the disc, the bobbins will be carried round with the laying block, but the strands will not untwist.

398. FIG. 2, PLATE XX, shows an epicyclic train which gives the same result as above when the pulleys *A* and *B*, which are here connected by a belt, are of the same size. This train has also been used in rope-making machinery, the pulleys *B* being often made slightly smaller in diameter than *A*, when it will be found that they will turn slowly in the opposite direction to that of the arm *D*, giving a slight additional twist to the strands of the rope as they are being laid, and making a rope less liable to twist.

✓ 399. FERGUSON'S PARADOX. By increasing the number of teeth in the last wheel of the three-wheel train (FIG. 5), it will be made to turn *R.H.*, and by decreasing the number, it will turn *L.H.* FIG. 9, PLATE XX, called *Ferguson's Paradox*, shows an arrangement giving both cases, the wheel *E* having the same number of teeth as *A*, *C* more than *A*, and *F* less than *A*, *B* being an idle wheel connecting *A* with the other three. The arm *D* turns freely on the axis of the stand *G*, while *A* is fast to the stand.

Representing the motions of each wheel for one *R.H.* turn of *D*, in tabular form, we have,

	A	*D*	*C*	*E*	*F*
Train locked	+1	+1	+1	+1	+1
Arm fixed	−1	0	$-\frac{60}{61}$	−1	$-\frac{60}{59}$
	0	+1	$+\frac{1}{61}$	0	$-\frac{1}{59}$

Thus *C* turns slowly *R.H.*, and *F* slowly *L.H.*, in respect to the wheel *E*, all gearing with the pinion *B*.

400. ADDITIONAL EXAMPLES. FIGS. 10 and 11, PLATE XX, show examples of epicyclic trains, in which annular wheels *C* and *E* are used. The extreme wheels are also upon the same axis, that of *A*, about which the arm *D* turns.

1. In FIG. 10 let the annular wheel *C* be fixed, and let the arm *D* make +10 turns; required the number of turns of *A* and *B*.

	A	*B*	*C*	*D*
Lock train	+10	+10	+10	+10
Fix arm	+30	−30	−10	0
	+40	−20	0	+10

Thus *A* makes +40 turns, and *B* −20 turns.

2. It is not necessary to consider the wheel *C* as fixed in this case, as we may allow it to turn in either direction while the arm turns, care being taken in the second operation of the table to have the resulting turns of *C* what they should be. Let the arm make +10 turns, and the wheel *C* −3 turns in the same time, required the number of turns of *A* and *B*.

We have the first line as before; but *C* has now been moved +10 turns; then, in order to have its resultant motion −3, we must, on fixing the arm, turn *C* −13 turns, which gives the second line of the table.

	A	*B*	*C*	*D*
Train locked	+10	+10	+10	+10
Arm fixed	+39	−39	−13	0
	+49	−29	− 3	+10

Then *A* make +49 turns, and *B* −29 turns.

3. In FIG. 11 let the arm *D* make +5 turns, and let *E* make +10 turns, find the turns of *B*, *C* and *A*.

	A	*B* and *C*	*D*	*E*
Lock train	+ 5	+ 5	+ 5	+ 5
Fix arm	—10	+15	0	+ 5
	— 5	+20	+ 5	+10

Then *A* makes —5 turns, and *B* and *C*, +20 turns.

401. Epicyclic bevel train. Fig. 12 represents an epicyclic train, made up of three bevel wheels. The shaft *SS* has a rigid arm *a*, whose axis is perpendicular to that of the shaft attached to it at *m*. The bevel wheels *c* and *d* turn loosely on the shaft *s*, but cannot move along it; the bevel wheel *e*, in gear with *c* and *d*, turns loosely on the arm between shoulders. In this train rotations will be considered + or —, as they appear when observed from the right hand end of the shaft *s*, + being to the right, [illegible] being to the left. This train is very generally used, and has very peculiar properties.

1. Let the arm *ma* be fixed, then we have a common bevel train, and *d* turns in an opposite direction to that of *c*. The wheel *e* transmits force from the pitch cone of *c* to that of *d* and the pressure on the bearing of the wheel *e*, which is resisted by the arm, is twice the force transmitted. By applying power to the gear *c*, and taking it from *d*, the arm *ma* being weighted to retain it in position, we can (knowing the number of rotations of *c*, and the weight on the lever with its position in respect to the axis of the shaft) determine the power transmitted. On this account this train has been used in some forms of dynamometers.

2. Let the wheel *c* be fixed, and suppose the arm to make +1 turn, then, proceeding as in previous cases, we have

	c	*arm*	*d*
Train locked	+1	+1	+1
Arm fixed	—1	0	+1
	0	+1	+2

Thus *d* makes +2 turns for +1 turn of the arm, or *d* turns twice as fast as the arm and in the same direction.

3. If we consider the wheel *d* as the driver, the arm will turn one-half as fast as it, and in the same direction.

4. Let the wheels *c* and *d* be given independent, equal, and opposite directions; then the arm *ma* will remain still, and *e* will

turn upon it. If c moves faster than d, then the arm will move in the direction of c, through one-half the angle gained; if c moves slower than d, the arm will move opposite to c through one-half the angle lost.

5. Let the arm ma, FIG. 12, be given a uniform rotation; then it will exert equal forces on c and d, and if the resistances are equal, these wheels will turn with the arm as though the wheels were locked. When the resistances are unequal, the wheel having the least resistance moves with the arm. This device has been applied in self-propelling agricultural engines where the power is applied to the arm, one of the traction wheels being attached to c and the other to d. In such case the train is constructed as shown in FIG. 13, where the gear wheel d, FIG. 13a turning loosely on the shaft, and carrying the bevels bb', takes the place of the arm, the bevel a, being attached by means of the shaft to one wheel, and c, by means of a hollow shaft turning loosely on the shaft SS, is attached to the other wheel.

When the engine comes to a curve, the outer wheel is allowed to move faster than the inner one, which has the most resistance, and thus no additional strain is brought to bear upon the axle. A similar device is applied to tricycles to make them curve easy.

402. GENERAL FORMULA of epicyclic trains having a uniform velocity ratio. Let FIG. 11 be taken to represent a general epicyclic train which may be considered as made up of four parts:

1. The wheel A;
2. The wheel E, both turning on the same axis,
3. The train arm D turning about the common axis of the wheels A and E,
4. A shifting train carried by the train arm.

The shifting train may consist of any pieces of mechanism employed to transmit a constant velocity ratio; such as spur, bevel, or skew-bevel wheels, circular pulleys and bands, links with parallel cranks, and double universal joints, etc.

Let the arm D make a turns, the first wheel A make m turns, and the last wheel E make n turns during the same time. Right-handed turns are to be considered $+$ and left-handed turns $-$. Also let e represent the value of the train, considering the train arm as fixed. This value is to be considered positve when the extreme wheels turn in the same direction, and negative when they turn in opposite directions.

Referring the motions of the wheels A and E to the arm D, the first, A, makes $(m-a)$ turns, and the last, E, makes $(n-a)$ turns relatively to the arm. Then we have for the value of the train.

$$e = \frac{n-a}{m-a}$$

which is the general formula for an epicyclic train.

We will now apply this formula to some of the proceeding cases.

The two-wheel train Fig. 6. Here $m = 0$, $a = 1$, and $e = \frac{T}{t}$; then substituting in the general formula, we have as there are two elements in the train

$$-e = -\frac{T}{t} = \frac{n-1}{0-1} \quad \therefore \quad n = 1 + \frac{T}{t}, \text{ if } T = t,\ n = 2$$

Fig. 7. Here $a = 1$, $e = -1$, $n = o$, then

$$-e = -1 = \frac{o-1}{m-1} \quad \therefore \quad m = 2.$$

The three-wheel train Fig. 5. Here $m = o$, $e = 1$, and $a = 1$; then

$$e = 1 = \frac{n-1}{o-1} \quad \therefore \quad n = o$$

Fig. 11. Here let D make $+5$ turns, and E $+10$ turns, to find m, the turns of A; then $a = 5$, $n = 10$, and $e = \frac{30 \times 25}{20 \times 75} = \frac{1}{2}$

$$\therefore \quad -e = -\frac{1}{2} = \frac{10-5}{m-5} \quad \therefore \quad m = -5.$$

Bevel train, Fig. 12, d to be considered the last wheel.

1. $a = 0$, $m = 1$, $n = -1$, and $e = -1$.
2. $a = 1$, $m = 0$, $e = -1 \ \therefore\ n = 2$.
3. $m = 0$, $n = 1$, $e = -1 \ \therefore\ a = \frac{1}{2}$.
4. First case, $m = -n$, $e = -1$. $\therefore\ a = 0$.

   Second case, $\frac{-n-a}{m-a} = -1 \quad \therefore\ a = \frac{m-n}{2}$

For $m>n$, a is positive.

For $m<n$, a is negative.

403 One of the most noted applications of epicyclic trains is in the controlling the motion of the cutting tool in boring cylinders. In the example (FIG. 15, ~~PLATE XX)~~ the cutters are attached to a frame which rides upon a massive cast-iron shaft or boring bar, and rotates with it; this frame is the recipient of a slow longitudinal movement given by a screw, on whose end is the pinion P FIG. 14. An annular wheel A, part only of whose teeth are shown, shaped as in the diagram, FIG. 14, PLATE XX, rides loose upon the bar, and drives the pinion P, whose pitch circle only is shown, at the end of the feed screw which advances the cutters. The boring bar is recessed as indicated to receive the screw. If the boring bar and the wheel A have the same angular velocity, the pinion P will not move at all; and further, a slow motion will be impressed upon P if the rotation of A be made to lag a little behind that of the boring bar.

A spur wheel B is keyed to the bar, a small shaft SS fixed at the side and passing through the bearing TT of the boring bar carries the wheels C and D. By some mechanism, not shown, the boring bar is caused to rotate, B being fast to the boring bar rotates with it, and thus through C and D, motion is imparted to the loose wheel A, and through it to the pinion P, and thence to the feed screw that advances the cutters. Let the numbers of teeth on the different wheels be as follows: B, 64, C, 36, D, 35, A, 64, both outside and inside, P, 16, and let the screw have a pitch of ½ inch.

$$\text{Then } e = \frac{B}{C}\times\frac{D}{A} = \frac{64}{36}\times\frac{35}{64} = \frac{35}{36}$$

That is A loses $\frac{1}{36}$ of a revolution for every complete rotation of the boring bar; or A makes $-\frac{1}{36}$ of a turn relatively to the bar.

Due to this falling behind of A the pinion moves $\frac{1}{36}\times\frac{64}{16}$ or $\frac{1}{36}\times\frac{A}{P} = \frac{1}{9}$ of a revolution, and as the pitch of the feed screw is $\frac{1}{2}$ an inch, the feed screw will advance the cutters $\frac{1}{9}\times\frac{1}{2}$ an inch, or $\frac{1}{18}$ an inch.

404. This slow motion of the feed screw which advances the cutters may be obtained in a more simple manner by an application of the sun and planet wheels of Watt.

In Fig. 16, Plate XX, the centre of motion of the combination *C* is at the centre of the boring bar, between the axes of the wheels *A* and *B*; *A* has 20 teeth, *B* has 90 teeth, and the value of e is $-\frac{2}{9}$. The wheel *B* is placed on the axis of the feed screw *X*, which advances the boring apparatus. The rotating arm *SS* is now fast to the solid part of the boring bar, and rotates with it due to some driving mechanism not shown. The arm *SS* carries the stud or axis of the wheel *A*.

The wheel *A* rides upon an axis that is carried by *SS* and is attached to a bar *D* of some convenient length which passes through and rests upon a fork, or has its lower end slotted, which slot moves up and down on a pin, similarly adjusted, so that the wheel *A* as it is carried around *B* by the rotation of *SS* is prevented from rotating by its connection with *D*, but is free to have its motion of translation about *C* by the possibility of oscillation in the rod *D*, due to its slot. In other words *A* is made a dead wheel, but can be carried around *B* by the rotation of *SS*. Due to this action of *A*, the wheel *B* will turn $\frac{2}{9}$ of a turn for 1 turn of *SS*, and if the pitch of the feed screw is $\frac{1}{4}$ of an inch, 1 turn of *SS* will advance the cutters $\frac{1}{4} \times \frac{2}{9}$ inches or $\frac{1}{18}$ an inch as before.

To reverse the feed, the stud or axle of *A* is adjustable, so that it can be lowered until *A* and *B* are no longer in gear, when the interposition of an idle wheel gearing with both *A* and *B* will accomplish the reversing of the direction of the feed screw, but will in no wise effect the amount of the feed, as it is the function of an idle wheel to change the directional relation of a train alone. The arm *SS* is provided with a slotted opening, *yy*, for the lowering of *A*, and the adjustment of the idle wheel.

V

PLATE XXI.

Differential Motions.

405. We have seen that an epicyclic train may also be formed by the use of three bevels, *e*, *c* and *d* (FIG. 12, PLATE XX) and that *c* and *d* turn in opposite directions, and that the arm *ma* carries *e*. The better way however of suspending *e* is to attach it to a spur wheel *dd* (FIGS. 13 and 13*a*, PLATE XX). In this system the wheel *a* is keyed to the shaft *SS*, *dd* is a loose wheel on the shaft *SS*, and receives an independent motion from some source; *c* may be a loose wheel on the main shaft, or keyed to a shaft that can rotate independently of the shaft carrying *a*. The wheels *bb'* are carried by *dd*, and gear with *a* and *c*. The wheel *c* is then the recipient of two independent motions that travel by different routes; one from the driving shaft through *a*, and the rotation of *b* and *b'* on their own axes, and the other due to the carrying of *b* and *b'* by the arm *d*, in its rotation about its axis. It is required to find the effect on *c* of varying the velocity of rotation of the arm *dd*.

As there are three wheels of equal dimensions in the train, viz: *a*, *b*, and *c*, the value of the train is expressed by the equation $e = -1$, or with the arm at rest relatively to *a*, or its effect being 0, *c* rotates with the same velocity as *a*, but in an opposite direction.

Let us apply the general formula for epicyclic trains, viz:

$$e = \frac{n-a}{m-a}.$$

1st. Let $a = m$, or let the arm *d*, turn at the same rate as *a*, then

$$e = -1 = \frac{n-a}{m-a}; \ -m + a = n - a, \text{ or } n + m = 2a = 2m, \text{ and}$$

$n = m$, or *c* has exactly the same motion as *a*.

2nd. Let $a = \frac{3}{4}m$, then

$e = -1 = \frac{n - \frac{8}{4}m}{m - \frac{3}{4}m}$; $\therefore\ n - \frac{8}{4}m = -\frac{1}{4}m$, and $n = \frac{m}{2}$, or c moves half as fast as a.

3rd. Let $a = \frac{m}{2}$, or the arm d turns half as fast as a, then

$$e = -1 \ \frac{n - \frac{m}{2}}{m - \frac{m}{2}} : \ \therefore\ \ n - \frac{m}{2} = -\frac{m}{2}, \text{ and } n = 0$$

or c does not turn at all.

4th. In general let $a = m - \frac{m}{x}$, then $e = -1 = \frac{n - a}{m - a}$;

$$\text{and } n = 2a - m = 2m - \frac{2m}{x} - m$$

$$\therefore\ \ n = m - \frac{2m}{x}$$

or when the velocity of the arm is made less than the velocity of the 1st wheel in the bevel train, the rate of diminution in the motion of the last wheel is twice that of the arm. Any reduction in the velocity of d, produces twice the reduction in the velocity of c.

406. The general way of imparting these independent motions to C is shown in FIG. 1, PLATE XXI. Suppose A, B, and C to represent three equal wheels, of which A is fast to the shaft DS, B is carried by the arm H, which is a loose wheel on DS, and C is a loose wheel. The shaft DS carries a conical pulley provided with grooves or steps at a, b, c, d, and e, where the diameters are 4, 5, 6, 7, and 8. EJ is a shaft parallel to DS, and carries a speed cone with grooves or steps whose diameters are the same as those of the cone on DS, but are in an inverse order. A wheel F is fast to this shaft EJ and gears with the arm H. The wheel F is half the diameter of H. The two cones are supposed to be connected by a crossed belt, and some source of power transmits a uniform velocity to the shaft DS, which is the driver of the system.

407. To find the circumstances of motion for C when H has its velocity varied, 1st, suppose the belt to be in the groove

marked *a;* then the diameters of the cones are as 4 to 8, or *EJ* rotates ½ as fast as the wheel *F*, with which it gears. So if *m* represent the number of rotations of *DS* and consequently of *A;* the velocity of the arm *H*, or

$$a = m\left(\frac{4}{8} \times \frac{F}{H}\right) = m\left(\frac{4}{8} \times \frac{1}{2}\right) = \frac{m}{4};$$

now in the epicyclic bevel train *A*, *B*, *C*, and *H*, *A* makes *m* turns, *H* makes $\frac{m}{4}$ turns, and the value of the train is —1; to find the number of turns of *C*. Substituting these values in the general formula we have

$$e = -1 = \frac{n - \frac{m}{4}}{m - \frac{m}{4}}; \quad \therefore \quad n = -\frac{m}{2},$$

or *C* turns half as fast as *A* and in the opposite direction.

2nd. Suppose the belt to be at *b*, then the velocity of *H* will be

$$a = m \times \left(\frac{5}{7} \times \frac{F}{H}\right) = m\left(\frac{5 \times 1}{7 \times 2}\right) = \frac{5}{14}\, m,$$

$$e = -1 = \frac{n - \frac{5}{14} m}{m - \frac{5}{14} m} \quad \therefore \quad n = -\tfrac{4}{14} m = -\tfrac{2}{7} m,$$

or *C* moves $\frac{2}{7}$ as fast as *A* and in the opposite direction.

3rd. Place the belt at *C*, then the velocity of *H* will be

$$a = \left(\frac{6}{6} \times \frac{F}{H}\right) m = \frac{m}{2}$$

$$\text{and } e = -1 = \frac{n - \frac{m}{2}}{m - \frac{m}{2}}; \quad \therefore \quad n = 0,$$

or *C* does not move at all.

4th. Place the belt at *d*, and we have the velocity of *H*

$$a = m\left(\frac{7}{5} \times \frac{F}{H}\right) = \frac{7}{10} m$$

and $e = -1 = \frac{n - \frac{7}{10}m}{m - \frac{7}{10}m}$; $\therefore$ $n = \frac{4}{10}m = \frac{2}{5}m$,

or C turns $\frac{2}{5}$ as fast af A and in the same direction.

408. FIG. 2, PLATE XXI, illustrates an application of the differential motion just described. *FF* is the shaft taking motion through the pulley *P* and a belt from the source of power. *A'* is fast to the shaft *FF*, as is *A*; *C* and *C'* are one piece and loose on the shaft, and *A*, *B*, and *C* are equal.

The combination of four bevel wheels here shown, differs in no respect from the bevel trains described. The two wheels *B* and *B* are equivalent to one wheel and prevent the unbalanced action that would otherwise occur. The shafts *SS* and *TT* carry the speed cones, and are connected by an open belt. The driving shaft *FF* is connected with the shaft *SS* by a train of 3 wheels, *L*, *M*, and *N*, *N* being ½ *L*; on the shaft *TT* is a pinion *E* gearing with *H*. *H* is five times the size of *E*, or the velocity ratio of *E* and *H* is $\frac{1}{5}$. Assume that the cones have equal diameters where the belt is placed. Then as the value of the train *N*, *M*, *L* is $e = \frac{1}{2}$, the cone on *SS*, and hence the cone on *TT*, and the pinion *E*, will move half as fast as the driving shaft *FF*, and in the same direction; and as *E* is $\frac{1}{5}$ of *H* and gears with it, *H* will turn $\frac{1}{10}$ as fast as the driving shaft *FF* and in the opposite direction.

Now to apply the formula;

$$e = -1 = \frac{n-a}{m-a},$$

since *A*, *B*, and *C* are equal, and $a = -\frac{m}{10}$; if m is the number of turns of *FF*, and therefore of *A*, then

$$e = -1 = \frac{n + \frac{m}{10}}{m + \frac{m}{10}}, \quad \therefore$$

n, or the number of turns of *C* and $C' = -\frac{12}{10}m$, or $n = -\frac{6}{5}m$.

Hence the speed of *C'* is to the speed of *A*, as 6 to 5. The negative sign showing, merely, that *C* and *C'* turn in opposite directions to the shaft *FF*.

409. By varying the position of the belt on the cones, varying differences of velocity may be obtained for *C* and *A*.

Let the belt be moved towards the gear wheels until the diameter of the cone on *SS* is to the diameter of the cone on *TT* as 2 to 3, then the velocity of *H*

$$= m(\tfrac{1}{2}\times\tfrac{2}{3}\times\tfrac{1}{5}) = \frac{m}{15}, \text{ or } a = -\frac{m}{15};$$

$$\text{also} \quad e = -1 = \frac{n + \frac{m}{15}}{m + \frac{m}{15}}, \text{ and } n = -\tfrac{17}{15}\,m;$$

or the velocity of *C* is to the velocity of *A* as 17 to 15. The use of this machine is as follows.

410. The operation of spinning consists in passing a partially twisted fibre called "roving" through a tube called a flier, attached to the end of the spindle, and thence to the spool or bobbin on which it is to be wound. Then a high velocity of rotation is given to the flier on the spindle and to the spool or bobbin. The motion of the fibre is uniform. If the flier or bobbin move at the same rate of rotation, the only effect would be to twist the roving, but they are made to rotate at different speeds, so that, the bobbin moving the faster, the fibre is twisted and wound on the bobbin at the same time.

411. This varying velocity ratio is accomplished by the differential mechanism shown in FIG. 2, where the wheel *C′* runs the mechanism that rotates the bobbins, and the wheel *A′* runs the mechanism that turns the spindle and fliers. This difference of velocities of rotation between the gear wheels *A* and *C* can be regulated by varying automatically, or at will, the position of the belt on the cone pulleys.

412. Given a differential motion shown in FIG. 3, PLATE XXI; *F* and *G* being connected by an open belt; *A*, *B*, and *C* have each 40 teeth, *D* has 100 teeth, and *E* 10 teeth; *F* is ten inches in diameter, and *G* is five inches in diameter.

1st. If the shaft *SS* makes 10 turns right-handed, how many turns does *A* make and in what direction?

2nd. If the belt connecting F and G were crossed, how would it effect the result?

In the first case the shaft makes 10 turns *R.H.*, therefore the arm D of the bevel train turns $10\left(\frac{10}{5}\times\frac{10}{100}\right)=-2$ times, as the belt is open, or expressed algebraically:

$$a=e\times 10\text{; and } e=-\frac{F}{G}\times\frac{E}{D}=-\tfrac{1}{5}$$

$$\text{and } a=-(10\times\tfrac{1}{5})=-2,$$

or D turns twice left-handed for 10 turns right-handed of S.

Then in the epicyclic bevel train $A\ B\ C\ D$, we have

$$e=-1,\ a=-2,\ m=10,\ \text{required } n.$$

In this case $e=-1=\frac{n+2}{10+2}$, $\therefore$ $n=-14$, or A turns 14 times left-handed for 10 turns right-handed of the shaft S.

2nd. The belt is crossed, and the velocity of D is

$$a=m\left(\tfrac{10}{5}\times\tfrac{10}{100}\right)=10\times\tfrac{1}{5}=2.$$

Therefore $e=-1=\frac{n-2}{m-2}=\frac{n-2}{8}$ $\therefore$ $n=-6$.

and A turns 6 times left-handed for 10 turns of S right-handed with the belt crossed.

413. Applying the tabular method in the first case, the process is the same as far as determining the turns of the arm D of the bevel train. Then the problem becomes the following, having given the epicyclic train A, B, C, and D; A, B, and C being equal, and C the driver, to determine the number and direction of the turns of A, when C turns 10 times *R.H.*, and D, the arm, turns 2 times *L.H.*

Suppose the entire system to be locked, and the system turned 10 times right-handed, then the elements of the train would all move together, and we should have the following tabular view,

A	B	C	D
$+10$	$+10$	$+10$	$+10$

but D was to turn 2 times left-handed, and it has been turned 10 times right-handed, therefore, to have the circumstances agree with the conditions imposed we must imagine D to be turned left-handed 12 times and change the balance to correspond, so we will have

	A	*B*	*C*	*D*
Train locked	+10	+10	+10	+10
Turning *D* 12 turns *L.H.*	—24			—12
	—14			— 2

In the second case the statement is the same only the arm *D* is to turn 2 times right-handed. Then with the train locked and the shaft *S* turned 10 times right-handed, we will have

	A	*B*	*C*	*D*
	+10	+10	+10	+10
Turning *D* 8 times *L.H.*	—16			— 8
	— 6			+ 2

which result correspond with the application of the general formula in either case.

PLATES XXII, XXIII AND XXIV.

414. The object of these plates is to exhibit various mechanisms and certain special contrivances, which are designed to accomplish certain special motions.

It is often required that a uniform circular motion of the driver shall produce an intermittent motion either circular or rectilinear, of the follower. The following examples will illustrate the manner in which this intermittent motion may be produced.

415. Fig. 1, Plate XXII, is one form of what is known in mechanism as the *Geneva Stop*. Its original use was in watches, to prevent overwinding of the spring. It is also used in various mechanisms to give a definite vibration to a piece from which the feed motion is derived.

Geneva stop. In Fig. 1, Plate XXII, the wheel A makes one-sixth of a revolution for one turn of the driver C, the pin b working in the slot b' causing the motion of A; while the circular portion of the driver, coming in contact with the corresponding circular hollows C', retains A in position when the tooth b is out of action. The wheel C is cut away just back of the pin b to provide clearance for the wheel A in its motion. If we close up one of the slots, it will be found that the shaft, a, can only make a little over five and one-half revolutions in either direction, before the pin, b, will strike the closed groove. This mechanism, when so modified, has been applied to watches to prevent overwinding, and is called the *Geneva Stop*, the wheel C being attached to the spring-shaft so as to turn with it, while A turns on the axis d in the spring-barrel. The number of slots in A depends upon the number of times it is desired to turn the spring-shaft.

By placing another pin, S, opposite b in the wheel C, as shown by dotted lines, and providing the necessary clearance, the wheel A could be moved through one-sixth of a turn for every one-half revolution of C.

416. Figs. 2 and 3, Plate XXII, show two methods of advancing the wheels *A* through a space corresponding to one tooth during a small part of a revolution of the shafts *C:* in this case the shafts are at right angles to each other. In Fig. 2 a raised circular ring, with a small spiral part, *b*, attached to a disc, is made use of; the circular part of the ring retains the wheel in position, while the spiral part gives it its motion. In Fig. 3 the disc is carried by the shaft, *CC*, and has a part of its edge bent helically at *b;* this helical part gives motion to the wheel, and the remaining part of the disc edge retains the wheel in position. By using a regular spiral, in Fig. 2, and one turn of a helix, in Fig. 3, the wheels *A* could be made to move uniformly through the space of one tooth during a uniform revolution of the shafts *C*.

417. The star wheel. In Fig. 4, Plate XXII, the wheel *A*, commonly known as the star wheel, turns through a space corresponding to one tooth for each revolution of the arm carrying the pin *b* and turning on the shaft *c*. The pin *b* is often stationary, and the star wheel is moved past it; the action is then evidently the same relative motion in regard to each other during the time of action. The star wheel is often used on moving parts of machines to actuate some feed mechanisms, and is analogous in its action to the *Geneva Stop.*

418. The contrivance known as a *masked weeel* enters into many ingenious mechanisms for numbering or counting. Its object is to enable a numbering machine to print the same number twice before the unit advances, as in numbering a cheque and its duplicate. The masked ratchet wheel consists of two ratchet wheels placed side by side, the teeth on one being of a uniform size, while the teeth on the other are alternately shallow and deep. The main feature is that the bottom of the cut in a shallow tooth is far enough from the centre of the shaft carrying the ratchet wheels, to cause the driver, or pawl, that moves the wheel with uniform teeth, to be so far removed from them, that the pawl, resting on the surface of a shallow tooth, does not touch the surface of the uniform teeth on the other wheel, and so does not move it. On the other hand when the pawl rests on the surface

of a deep tooth, it can engage the uniform teeth on the other wheel and thus drive both wheels. The two ratchet wheels ride upon the same axis, but move independently of each other, the uniform ratchet wheel being connected with the numbering apparatus.

419. FIG. 5, PLATE XXII, illustrates the action of a masked wheel; the pin wheel *D* represents the first ratchet wheel, and is fast to the axis *a;* the second wheel *A* has its teeth arranged in pairs every alternate tooth being cut deeper, and it turns loosely on the axis *a*. The click *B* is so made that one of its acting surfaces, *i*, bears against the pins, *e*, of the wheel *D*, while the other face, *g*, is placed so as to clear the pins and yet bear upon the teeth of *A*, the wheel *A* being located so as to admit of this.

If we now suppose the lever, *C*, to vibrate through an angle sufficient to move either wheel along one tooth, both having the same number, it will be noticed that when the projecting piece, *g*, is resting in a shallow tooth of the wheel *A*, the acting surface, *i*, will be retained too far from the axis to act upon the pin, *e*, and thus the vibration of the lever will have no effect upon the pin wheel, *D*, while when the piece, *g*, rests in a deep tooth, as *b'*, the click will be allowed to drop, so as to bring the surface, *i*, in action with the pin, *e*.

In the figure the click, *B*, has just pushed the tooth, *e*, into its present position, the projection, *g*, having rested in the deep tooth, *b'*, of the wheel, *A;* on moving back, *g* has slipped into the shallow tooth, *b*, and thus the next stroke of the lever and click will remain inoperative on the wheel, *D*, which advances but one tooth for every two complete oscillations of the lever, *C*.

Both wheels should be provided with retaining pawls, one of which, *p*, is shown. This form of pawl, consisting of a roller, *p*, turning about an axis carried by the spring, *s*, attached to the frame carrying the mechanism, is often used in connection with pin wheels, as by rolling between the teeth it always retains them in the same position relative to the axis of the roller; a triangular pointed pawl, *F*, pivoted at *E*, which also passes between the pins is sometimes used in place of the roller.

The pins of the wheel, *D*, might be replaced by teeth so made that their points would be just inside of the bottoms of the shallow teeth of *A* ; a wide pawl would then be used, and when it rested in a shallow tooth of *A*, it would remain inoperative on *D*, while when it rested in a deep tooth it would come in contact with the adjacent tooth of *D* and push it along.

So long as the click, *B*, and the wheels have the proper relative motion it makes no difference which we consider as fixed, as the action will be the same whether we consider the axis of the wheels as fixed and the click to move, or the click to be fixed and the axis to have the proper relative motion in regard to it. The latter method is made use of in some forms of numbering machines.

420. As an illustration of the use of the pin wheel and a single toothed wheel, attention is called to one of the earliest types of counting wheels for registering the number of revolutions of any shaft with which it may be connected.

Each wheel of a series *A*, *B*, *C*, Fig. 6, Plate XXII, possesses 10 pins on its face, and it is so arranged, that a single tooth on each wheel shall be sufficiently long to reach the pins on the wheel to its left. These teeth are shown at *P* on the face of *C*, and at *Q* on the rear of *B* so as not to catch on the pins of *C;* the pins of *A* project to the rear to gear with *Q*. The tooth on *C*, marked *P*, drives the wheel *B*, and *Q* drives *A*.

Thus *C* goes round ten times while *B* makes one revolution, and so on for the other wheels. In this way the series is capable of counting units, tens, hundreds, thousands, etc.

The top figures indicate the count, as 988 in the figure. To this end the front of the mechanism is masked by a plate, with small apertures corresponding to the present positions of 9, 8 and 8, on *A*, *B*, and *C*, respectively, so that only the counting numbers show ; attention is called to the position of the tooth *P* with reference to the numbers on the wheel *C;* it is so located that when the 0 shows at the aperture, it shall be clear of the pins on *B*. Rotation is imparted to the wheel, *C*, by means of the vibrating arm, *EF*, and the pushing pawl, *HD*. The arm, *EF*, is one element of a linkage mechanism, the corresponding and opposite link being a rotating crank on the shaft, the number of whose rotations is desired. The number now registered is 988, after two vibrations of the arm, *EF*, the wheel, *C*, will be advanced two teeth or pins and the zero of *C* will be at the highest point and shown through the aperture, and the tooth, *P*, will advance the wheel, *B*, one tooth or pin, and the number 9 will be at the highest point of the wheel, *B*, and show through its aperture, and the number registered will be 990; after ten more vibrations of the arm, *EF*, the tooth, *P*, will again advance the wheel, *B*, one pin, and at the same time the tooth, *Q*, will advance the wheel, *A*, one tooth or pin in the direction of the arrow, and the three wheels will now register 000, having passed 999, which is the highest number this combination can give us. The wheels, *A*, *B*, and *C* are retained in position by the rollers, *R*, *R*, *R*, mounted on springs, *T*, *T*, *T*. As each roller is forced in between two pins, it not only acts as a detent, but also adjusts the wheel in its right position. A small counting apparatus used on gas meters consists of a series of dial plates fixed in their position and a separate pointer travelling around each dial plate respectively. The step by step, or intermittent motion is not employed, but each pointer is on the axis of the elements of a train of wheels composed of a wheel and pinion upon each axis, the number of teeth on any wheel being ten times that upon the pinion which drives it. So that if the pointer on the dial plate registering thousands completes a revolution, it adds ten thousand to the register, and so advances the pointer on the dial plate, that registers tens of thousands, over one division, and an inspection of the pointers throughout the series will give the reading. These successive pointers move in opposite directions, as they are attached to the successive axes of a train of wheels, and these successive axes move in opposite directions, as only two wheels are thereby made to gear; also the figures on the successive dial plates are in reverse order to correspond with the motions of their respective counters. The train of wheels is moved by the constant rotation of an endless screw, this rotation being induced by the flow of gas.

421. A reliable counting apparatus is derived from the combination of a ratchet wheel with the train of wheels used in the gas meter; the pawl that moves the ratchet wheel being connected

with a vibrating arm that is connected with the particular mechanism whose motions it is desired to register.

The axis of the ratchet wheel carries a pinion of 10 teeth, gearing with a wheel of 100 teeth on an axis that carries a wheel of 10 teeth driving another wheel of 100 teeth, and so on throughout the series.

This train cannot fail to record the tens, hundreds, thousands, etc., accurately; the only possibility of a mistake is in the units, but if the step by step movement be masked, and the ratchet wheel driven a little beyond the point necessary to give another unit, or if the step by step movement be a little overpronounced the register will be perfectly correct.

To avoid the objectionable feature of figures on successive dial plates being in reverse order, and successive pointers moving in opposite directions, idle wheels can be inserted between the successive axes, and all the dials can then read the same way and the pointers can all move in the same direction.

422. In mechanism, *the fusee* is frequently employed to transmit motion, and then it enables us to derive a continually increasing or decreasing motion from the uniform motion of the fusee shaft.

The groove of the fusee may be traced upon a cone or other tapering surface, or it may be compressed into a flat, spiral curve; in all cases the effect produced will be that due to a succession of arms which radiate in perpendicular directions from the fixed axis, and continually increase or decrease in length.

Two fusees may be combined so that the motion produced may be increasing at first and then decreasing at the last. Such a device, known as a scroll, is used to operate the carriage of a spinning mule which should have an accelerated motion up to the middle of its path, and then a retarded motion to the end of its path, the start and stop being slow.

The fusee is adopted in chronometers and watches, in order to maintain a uniform force upon the train of wheels, and to compensate for the decreasing power of the spring.

423. FIG. 1, PLATE XXIII. The spring is enclosed in a cylindrical barrel, *B*, and sets the wheels in motion by the aid of a cord,

C, or chain wound partly upon the barrel and partly upon a sort of tapering drum called a *fusee*.

As the spring uncoils in the barrel the pull of the cord decreases in intensity, at the same time, however, the cord unwinds itself from the fusee and continually exerts its strain at a greater distance from the axis *A* and consequently with greater leverage and greater effect. Hooke's law of extension and force determines the circumstances of the varying tension of the spring and the consequent necessary cross section of the fusee.

424. HOOKE'S LAW states that when a body is compressed or extended in a degree less than that which produces a permanent change of form, the force necessary to keep it in this state of compression or extension is proportional to such compression or extension. As an example take the case of a spring balance; hang on it, successive weights of 1, (2), (2+1) (3+1) pounds, the index point will descend through equal spaces for each additional pound weight, and will rise by equal spaces as each pound is successively removed. Assuming this law to hold exactly when the spiral spring is subjected to a force of torsion in place of one of direct extension, we shall have from Hooke's law, that the pull of the spring is proportional to the angle through which the barrel has been made to turn.

425. In FIG. 2, PLATE XXIII, let *DPBA* represent one-half of the section of a fusee, *DPB* being the curve whose equation we desire to find. Draw *DE*, *PN*, *AB*, all perpendicular to *EA*; make *ER*, *QN*, *SA* represent the pull of the spring when the chain is at the points *D*, *P* and *B*, respectively. According to Hooke's law, the pull of the spring will decrease uniformly as the chain passes from *D* to *B* on the surface of the fusee, therefore the line joining *R*, *Q*, *S* and any other points measuring the pull of the spring must be in a straight line, inclined to *EA*. Produce this straight line *RQS* to meet *EA* in *C*. By reason of the law of elasticity, $QN : SA :: CN : CA$, and $\frac{QN}{CN}=\frac{SA}{CA}$, assuming that this is represented by m, we have $QN = m \times CN$. In order that the fusee may accomplish its object, and preserve a uniform tension on the main spring, the product on the pull of the spring into the lever arm, furnished by the curve of cross

section of the fusee, must be constant for every position of the chain on the fusee; take any point, as P, NP is the lever arm of the spring referred to and QN is the measure of the pull of the spring, then

$$[\text{The pull of the Spring}] \times [\text{Lever Arm}] = QN \times NP,$$

or since $QN = m \times CN$,

$$\text{Pull of Spring} \times NP = m \times CN \times NP.$$

Let $CN = x$, and $NP = y$, then

$$\text{Pull of Spring} \times NP = mxy$$

or according to the desired object of the fusee,

$$mxy = \text{a constant},$$

$$xy = \text{a constant},$$

which is the equation of a hyperbola.

426. In practice where great accuracy is required, the strength of the spring is tested by fixing a light lever to the winding square of the fusee, and observing whether the pull of the spring is balanced in every position by the same weight hung at the end of the lever. Then if the pull of the spring is not so balanced the surface of the fusee is cut away, or altered, so as to accomplish the perfect balancing of the pull of the spring in every position of the chain, by the same weight.

427. In mechanism the fusee is frequently used to transmit motion instead of to equalize force, and enables us to derive a continually increasing, or decreasing circular motion from the uniform rotation of the driving shaft. The groove of the fusee may be either traced on the surface of a cone or other tapering surface or the horizontal projection of this groove may be used, in the form of the compression of the tapering surface and the fusee groove, into a flat spiral curve; in all cases the effect produced will be that due to a succession of arms which radiate from a given axis, and continually increases or decreases in length. The fusee can only make a limited number of turns in one direction.

428. A flat spiral fusee occurs in spinning machinery and serves to regulate the velocity of the spindles, and to insure the due winding of the thread in a succession of conical layers. The winding on of the thread begins at the base of the cone and proceeds upwards towards the vertex; the spindles are driven by a

drum which rotates under the pull of a chain, and they may be made to revolve with increasing rapidity by placing a fusee upon the driving shaft and causing the chain to coil thereon. Such an arrangement as is shown in FIG. 3, PLATE XXIII, will be adapted to the winding on of a uniform supply of thread, with a uniform tension, on a conical surface. The chain there shown drives the driver that drives the spindles. FIG. 4, PLATE XXIII, shows a method of contracting the form of the fusee so as to vary the rate of increase. It consists of two flat discs having axes at *A* and *B*, and upon which are cut radial and curved grooves respectively in the manner indicated, it being arranged that when one plate is placed upon the other, the pins, *P* and *Q*, shall travel in both sets of grooves and at the same time. As the blocks that carry the pins, *P* and *Q*, will move along the radial grooves as the disc, *B*, turns relatively to *A*, this combination will form a spiral fusee of any required form and its dimensions can be enlarged or contracted at pleasure. It is an example of aggregate combination wherein the composition of two uniform components gives a variable resultant motion.

429. FIG. 5, PLATE XXIII. If two cords be placed in opposite directions around a drum, *A*, and the ends of the cords be fastened to a movable carriage, the rotation of *A* in alternate directions will cause a uniform reciprocating movement in the carriage. If the drum in place of being cylindrical, were formed of two fusees on tapering surfaces, placed base to base, as in FIG. 6, PLATE XXIII, and the cord attached as before, we should have for the rotation of the fusee in alternate directions, a reciprocation of the attached piece, with a variable motion.

Hence the drum, *A*, FIG. 5, PLATE XXIII, has been replaced by a spiral fusee in certain spinning mechanism, where variable motion is necessary and thus the motion of the reciprocating piece is gradually accelerated, until it has reached the middle of its path, and then decreases to the end of the movement. The cord fastened at *A* leaves the fusee at *C*, and that of *B* leaves at *D*.

430. A helical groove of varying pitch, traced upon the surface of a cylinder, would produce a similar variable motion by the rotation of the cylinder on its axis, in a reciprocating piece that

connects with the cylinder by means of a block or pin free to move in the groove. This mechanism is a substitute for the fusee.

431. The mechanism shown in Fig. 7, Plate XXIII is based on the principle of the fusee. A variable winding on motion in spinning is its object.

Let one end of a rope which is coiled around a drum be attached to a point, P, in the movable arm CP. Then the rotation of the arm, CP, about its centre, C, will cause some rope to be unwound from the drum. Draw CS perpendicular to PQ; then at any instant of motion the rate at which the rope is unwound from the drum is proportional to the lever arm of the pull of the rope, that is to CS. This is in effect a four bar linkage, $CPBQ$, and the rate of unwinding is greatest when CP is perpendicular to PQ, but decreases to zero, when CS becomes zero, or when CP coincides with PQ at the dead points, and consequently the varying arm of the fusee exists.

By varying the point of attachment of PQ and PC, or by making P movable, the rate of variation in the motion of the drum can be varied at will. Any mechanism attached to the drum, or that receives a motion of rotation from the rotation of the drum, as the spindles in spinning, can be made the recipient of a variable motion that can be adjusted so as to wind on a conical surface with uniform tension.

Mangle Wheels.

432. Mangle wheels are used to produce reciprocating motion from the uniform rotation of a pinion, and derive their name from the first machine in which they were applied.

Figs. 8, 9, and 10, Plate XXIII, show three forms of mangle wheels. In Fig. 8 the teeth are only drawn in on a part of the pitch curve pp, and in Fig. 9 the pitch curves only are shown.

In Figs. 8 and 9 the cycle of motion of the wheel is divided into two parts, each part having its own definite velocity ratio which is here constant except for a small space at each end of the motion, when the pinion is being guided from one pitch circle, *p*, to the other, they being joined at their ends by short circular arcs.

433. FIG. 8 has the teeth cut upon the edge of an annular groove in the disc *A*, these teeth being properly formed to gear with the pinion, *P*, the shaft of which has sufficient motion, on a line through *C*, to allow the pinion to gear with both the inner and outer sides of the groove. At the same time the pinion's shaft projects below the pinion, and works in the groove *aa*, shown black, the width of the groove being a little greater than the diameter of the pinion's shaft. This groove serves to always keep the pinion in gear. If we suppose the pinion to rotate right-handed, the wheel, *A*, will first make about ¾ of a rotation left-handed, and then about one rotation right-handed, and so on. It will be noticed that the change of motion is gradual at each end, when the pinion is passing from one position to the other.

434. In FIG. 9, the wheel, *A*, has teeth cut upon the outside of an annular ring projecting from the face of *A*, the pinion now travelling outside of the pitch circle, *pp*, a groove being supplied for the shaft as before. Here the difference between the velocity ratios is less than in the previous case, and by making the two pitch lines, *pp*, to coincide, as has been done in FIG. 10 the velocity ratios are made the same.

435. In FIG. 10, the wheel, *A*, is supplied with a series of pins, *p*, the pinion, *P*, working alternately on the inside, and then on the outside of the pins. One method of connecting the pinion to a fixed, upright shaft, *c*, is also shown. The arm, *b*, turning on the shaft, *c*, carries the pinion shaft, *s*, and a train of gear wheels, *e*, *f*, *g*, connects the shafts, *s* and *c*; the gear wheels, *e* and *f*, are shown dotted. The shaft, *s*, projects below the pinion, and can only move between the positions, *s* and *s'*; the pieces, *GG*, against which it comes when the last pin is reached serve to guide it from one side of the teeth to the other. As the teeth of the pinion cannot be shaped to give an exact velocity ratio, and at the same time work both on the inside and outside of the pins, this arrangement cannot be used where great uniformity of motion is desired.

The wheels illustrated have their teeth arranged in circular rows, but this is not necessary, as by properly curving the rows of teeth the velocity ratio may be varied at pleasure; a pause can also be introduced by making the row radial for the corresponding distance.

436. Figs. 11 and 12, Plate XXIII, show two forms of mangle racks, which are lettered the same as the wheels, the shafts of the pinions being constrained to move in lines, parallel to the pitch line.

In Fig. 12 the pinion *P* will gear correctly with the circular pins, and in both cases the velocity ratios will be the same.

Sometimes the pinion is fixed, and the rack shifts laterally, it being arranged to move in suitable guides or to be guided by linkage properly arranged.

Mangle racks are used in some forms of cylinder printing presses to actuate the table, see Fig. 13, Plate XXIII.

437. Fig. 13, Plate XXIII illustrates a use of movable rack and fixed pinion. The rack *HF* is attached to the system of levers as indicated in the diagram. It will be observed that *A* and *C* are centres of motion, and that *B* is the centre of *AG*, and *D* is the centre of *CE*, and that *ABDC* is a parallel motion, so that when the pinion *P* has pushed the rack *HF* to either end of its path, the rack will be moved to the other side of the pinion, the parallel motion constraining it to maintain a direction coincident with that of the reciprocating piece and always parallel to its original direction in all of its positions.

The reciprocating piece is a table in a printing machine. To this table are attached the centres of motion, *A* and *C*, of the linkage *ABDC*, that furnish the parallel motion.

Thus the table carrying the fixed points of the parallel motion and likewise the rack *HF* by means of its attachment to the parallel motion through *GH* and *EF*, oscillates backwards and forwards, while the pinion *P* which transmits the force, that causes this desired reciprocation, remains fixed in place. Guides are furnished for the rack and pinion.

438. The disc and roller.—This arrangement consists of a disc *D*, Fig. 1, Plate XXIV, rotating about an axis perpendicular to its plane and giving motion to a friction roller *a*, turning upon an axis perpendicular to and intersecting that of *D*.

If we suppose the rotation of the disc to be uniform, the velocity ratio between *a* and *D*, will constantly decrease as the roller *a* is shifted nearer the axis of *D*, and conversely.

If the roller is carried to the other side of the axis *d*, as to *b*, it will rotate in the opposite direction to the first.

By changing the distance of the roller from the axis, as the disc revolves, the action of the fusee may be imitated. The disc might be supplied in this case with a spiral row of teeth, in which a pinion, taking the place of the roller *a*, might be made to gear.

This combination is sometimes used in feed mechanisms for machine tools, where it enables the feed to be adjusted and also reversed by simply adjusting the roller on the shaft *cc*.

439. FIG. 2, PLATE XXIV, shows a disc and roller with a spiral rack. As the disc revolves, the pinion *P* slides upon its square shaft, and is kept on the rack by the action of a guide roller *R*, which travels in a groove arranged for the purpose.

As an exact mechanism this combination is deficient, in that the two edges of the roller or pinion, move at the same velocity although at different distances from their centre of motion *C*; this is an impossibility, and the two edges cannot roll together, so there must be sliding motion. The mechanism does furnish the principle for a machine for grinding or pulverizing. Two rollers running around upon a flat surface or bed stone, in the manner suggested, form the combination. These rollers are called edge runners; they are of large size and very heavy and are placed near the vertical axis about which they roll so as to reduce the inequality of motion to a minimum.

By inspection of FIGS. 3 AND 4, PLATE XXIV, we see that if *B* is the driver, that a continuous rotation of *B*, with a reciprocating motion of translation, above and below *C*, will give a reciprocating motion of rotation to *A*, and conversely, if *A* is the driver, and *B* has this reciprocating motion of translation above and below *C* that a reciprocating motion of rotation of *A*, with periods corresponding to the translation of *B* above and below *C*, will produce a motion of rotation of *B* in one direction, which direction remains invariable.

440. This movement has been applied in the construction of a continuous indicator, Fig. 5, Plate XXIV. In this apparatus, the piston of the indicator is attached to the roller *B*, and receives the pressure of the acting steam in the engine cylinder, and the plate *A* takes up, on a diminished scale, the reciprocating motion of the piston of the engine. A grooved wheel *M* is connected directly with the disc *A* and receives a reciprocating motion, by means of a cord passing over the wheel and attached to the piston rod.

The roller *B*, connected with the indicator piston, travels up and down with this piston, in the line *PE*, being subject to the opposing forces of a steel spring that forces the roller downwards, and the pressure of the steam in the engine cylinder, which steam having access, by construction, to the indicator cylinder, tends to raise the indicator piston against the spring, and with it the roller *B*, which can thereby pass and repass the centre of *A*, having its direction of rotation changed if *A* remains constant, or unchanged if *A* changes. The axis *EP* is provided with an elongated or broad pinion *L*, to gear always with *H;* the axis of *H* carries at the top of the line *ba*, the recording or counting apparatus. The principle relied on is, that the work done by the engine, in a given time, will be proportioned to the number of revolutions of *H*. In the line *PE* is a small steam cylinder and piston connected with the engine cylinder, this is the indicator cylinder referred to, and the indicator piston is subject to the pressure of the steam in the cylinder, and by reason of the strength of the spring referred to being known, the value of the vertical movements of the indicator piston are known. This piston carries the roller *B* and the broad pinion *L*. When there is no steam pressure acting on the indicator piston the disc *B* is at the centre of the plate *A*.

Any increase of steam pressure carries *B* above the centre of *A* and causes *H* to rotate with a velocity that increases as the steam pressure rises. Should *B* cross the centre of *A* before the direction of rotation of *A* is reversed, *H* will begin to rotate in the reverse direction, and the register will record a subtractive quantity of work, and the instrument may be said to take the algebraic sum of the work done, throughout each stroke, for any interval of time.

441. One application of the use of epicyclic trains is to make the train arm move, as in some governors for purposes of regulation, with a velocity proportional to the difference between the velocities of the primary wheels. This is readily effected in the epicyclic bevel train. See FIG. 6, PLATE XXIV, where the wheels B and C are made to rotate in contrary directions, and they are so connected by a shifting train, that when the train arm is at rest, the angular velocities of the primary wheels are equal and opposite. In the figure, O is the vertical spindle carrying a tight pulley B', that, by means of a cord and groove, is made to impart the velocity of the driving shaft to the spindle O, in a given ratio. O carries the bevel wheel B, which is part of B', and the bevel wheel C, and these wheels turn in opposite directions as part of the bevel train CFB. A is the train arm, being a horizontal spindle, carried by a collar which turns about the vertical spindle.

The shifting train consists of a bevel train turning about the train arm A as a spindle, and gearing with B and C. The result is, that when the velocities of B and C become unequal, the train arm turns in the direction of the greater of the two velocities, with a speed equal to half their difference. In order that C can move with a greater velocity than B it must be the recipient of some other component motion than that which comes through the bevel train BFC, else its velocity will always equal that of B. In practice some weight or resistance is applied to C, which furnished a parallel to the differential motion described.

442. FIG. 7, PLATE XXIV, shows a combination that produces this result. It is the application of an epicyclic train to a conical pendulum. An epicyclic train of three equal bevel wheels A, B, and C, is placed between the driving power and the pendulum. A is driven by the engine, C is connected with the pendulum, and B is the shifting train whose motion around A and C is limited by stops. The joints at F and B are constructed so as to permit this movement. The wheel B is also connected by a system of levers, FEL being a bell crank lever, with a weight K, and the steam valve is closed when K is lifted through a certain

distance. The valve spindle passes through *E*, and any motion of *FE*, caused by the raising of *K*, tends to close the valve. The conical pendulum *DP* is suspended by a ball and socket joint at *S*, and the extremity *D* moves in a circular slot *DH*, so that *DP* takes up the rotation of *C*. A certain amount of energy is absorbed in preserving the pendulum at a constant inclination to the vertical, and this, with the friction that opposes the motion of the pendulum, furnishes the resistance that produces the inequality in the velocities of *A* and *C*, and causes the motion of translation of *B* about the spindle *A′*. So the tendency of *C* is to lag behind *A*, and the governor begins to work as soon as *K* is raised through any appreciable distance, and any tendency to an increase of velocity in *A* produces a corresponding change of position of *B* and a similar raising of *K* which controls the closing of the steam valve. So sensitive is this combination to fluctuations in the speed of its elements, that an alteration of $\frac{1}{50}$th of a revolution may close the valve.

PLATE XXV.

Engaging, Disengaging and Reversing Gear.

443. We now take up a class of mechanisms that have for their object the transmission of a motion of rotation from one shaft to another, whose axis is coincident with the first, or for reversing the direction of rotation of a given shaft. These mechanisms are called *engaging or disengaging and reversing gear.*

444. Engaging and disengaging gear or disengaging and reengaging gear, is the name given to those mechanical contrivances by means of which the connection between a follower and driver can be begun and stopped at will, that is by means of which the combination can be thrown into gear and out of gear when required. As the eccentric bearing or longitudinal sliding of the back gear of a lathe.

445. Disengagements, as these contrivances may be called, may be classed in different ways, as those that act by pressure, and those that act by friction. Those which act by pressure are ~~precise~~ and definite in their action, as the connection between the pieces that are thrown in or out of gear at a given instant is established or broken at once, in a certain definite position of the pieces, and with a certain definite velocity ratio. As when a connection is made or broken between gear wheels or toothed clutches.

446. Disengagements which act by friction are to a certain extent indefinite in their action, the change is produced by degrees, and consequently the relative position of the pieces, when the connection is completely established or broken, is uncertain. In some cases the definite action of the first class is necessary; where it is unnecessary, the second class have a great advantage, in the avoidance of shocks and strains that must obtain with the use of the first class. The practical classification of disengagements is as follows:

1st. Disengagements by means of couplings, where two pieces that turn about one axis are coupled or uncoupled at pleasure, so that when coupled, they turn as one piece; these may transmit motion either by friction, as with concentric, conical or cylindrical surfaces, or by pressure as in toothed clutches.

2nd. Disengagements with rolling contact. These always transmit motion by friction; example, grooved friction gearing where one of a pair of axes so connected, is movable, and contact between the wheels can be made or broken at will.

3rd. Disengagements with sliding contact. These always transmit motion by pressure, and in most cases they act by throwing toothed wheels or screws in or out of gear.

4th. Disengagements by shifting belts; as the combination of a loose, and idle pulley, and belt connecting two shafts.

447. Reversing gear usually consist of a double set of engaging and disengaging gear, by means of which the follower can be thrown into gearing with either one of the two drivers, that drive it in opposite directions, or it may be disengaged from both, as the combination of a broad pulley, two idle pulleys and a fast pulley and a crossed and an open belt connecting two shafts, one of which carries the broad pulley, the other the fast and two loose pulleys.

448. 1ST CLASS.—A clutch is a sort of coupling in which one rotating piece drives another piece that turns about the same axis, by means of two or more projections that fit into corresponding recesses in the second piece, or the bearing surfaces that are to be brought into contact, may be cut into symmetrical saw tooth surfaces that shall fit together accurately, FIG. 3, PLATE XXV, whereby the rotation of the driver can be imparted to the follower by bringing these toothed surfaces into close contact. In a disengaging clutch the driving piece is a cylindrical box or collar with suitable projections, as in FIG. 2, PLATE XXV, which can slide on the rotating shaft, but which turns with the shaft, which is splined or grooved for this purpose, and has a keyed connection. On the other surface of the driving part of the clutch is a groove to carry the forks of the lever that gives motion to it, sufficient to engage or disengage the projections on the driver and the corresponding recesses in the follower. Motion may be transmitted by these means to another length of shaft

turning about this same axis, or to a loose wheel or pulley. Or the power may be received from the source by the loose pulley and by means of the clutch imparted to the shaft that carries it. Sometimes the acting faces of the clutch instead of being planes perpendicular to the axis of rotation, are inclined backwards, as regards the direction of motion, at an angle of about 15°, so that some forward pressure must be continually exerted on the clutch when in gear to make it maintain the connection.

Then any sudden change in velocity of one of the parts of the coupling, breaks the connection and prevents the transmission of shocks to the followers.

449. In the friction clutch, the following piece is a circular disc, having a hoop which surrounds it, and which can be so adjusted in its bearing on the disc by screws, that there shall be just enough friction between the hoop and the disc to transmit the motion of the hoop to the disc. This hoop has a clutch connection with a sliding piece, that has spline and key connection with, and rotates with, the driving shaft, and which can be brought in or out of gear with the hoop by means of projections and corresponding recesses, or their equivalent, so that the velocity of the sliding piece or clutch can be at once imparted to the hoop; but the hoop at first slips on the disc and only gradually does the disc take up the motion of the hoop and thus dangerous shocks are avoided.

450. In the friction cones the driver may be a conical sliding piece, with spline and key connection with the shaft, and turns with it, as *D*, FIG. 4, PLATE XXV.

The follower has a projecting rim, the inner surface of which is turned to a very accurate and smooth conical surface, and is loose on the shaft, and fast to a pulley, or fast to a secondary piece of shafting to which we wish to impart rotation, as *P* FIG. 4.

The surface of the follower is turned to a conical surface exactly fitting that of the driver. Or the outer surface of the driver may be made to fit the inner surface of the follower, in either case these surfaces are concentric conical surfaces accurately turned, as in FIG. 4.

451. FIG. 4, PLATE XXV.—The driver is moved along its shaft by an attached lever, and when the conical surfaces referred to,

are brought in contact, the motion of the driver is imparted to the follower. The angle of obliquity of the conical surfaces should be just great enough to prevent any risk of jamming, and for that purpose an angle of about 10° is sufficient.

452. In the frictional sector coupling, the follower is a cylinder with projecting rim, the inner surface of which is turned true and smooth.

The driver is a smaller cylinder fast to the shaft and turning with it. It carries an expanding disc, composed of two movable sectors of a cylinder concentric with the first, with true and cylindrical smooth rims, fitting exactly the inner surface of the rim of the follower. These sectors can be simultaneously moved to and from the axis of the shaft by means of inclined surfaces or wedges, that are worked by links and levers, attached to a movable collar, that slides on the shaft.

453. Fig. 5, Plate XXV.—This collar is moved in the direction of the axis of the shaft by means of a forked lever.

Movement in one direction forces the sectors from the shaft against the rim of the follower; movement in the opposite direction breaks the connection between the rims of the driver and follower. When the sectors are moved from the axis of the shaft, they fit tightly to the inside of the hoop shaped rim of the follower, and by their friction, communicate this rotation to the shaft.

When the sectors are moved towards the centre, the connection is broken, and disengagement takes place.

454. In all these cases, a loose pulley and the stationary part of the clutch, may be the driver. As it is often the case, that it is not desired to have the main shaft turning at all times, and the engine, furnishing the power, may be required to run constantly for other purposes, a loose pulley on the main shaft, receives the belt from the driving pulley of the engine, and the velocity of rotation is due to the ratio of these pulleys; then any of the clutches described, but preferably the frictional cones and cylinders, will furnish the engaging and disengaging gear necessary to turn the shaft at will, or permit it to remain at rest.

455. Class II.—The principle of the method of effecting engagement and disengagement by wheels in rolling contact is

shown in Fig. (9, Plate XXV, *A* and *B* are the axes of a pair of smooth wheels whose surfaces do not touch each other, *A* being the driver and *B* the follower. *C* is the movable axis of an intermediate idle wheel which drives *B* and is driven by *A*; *D* is the pitch point of *A* and *C*, and *E* is the pitch point of *C* and *B*. the straight line *DE* is the common line of connection of the three wheels, and as the pressure only and not tension can be transmitted along that line from the first to the third wheel, the connection ceases if the motion is reversed.

456. This would not, however, be the case with toothed wheels. To disengage the wheels while in motion forwards, the axis *C* is shifted so as to break the contact at *D* or *E* or at both points. The angle of obliquity, $CDE = CED$, which the line of connection *DE* makes with the two lines of centres, *AC* and *BC*, ought to be a little greater that the angle of repose of the surfaces of the wheels, in order that the wheel *C* may not become jammed between the wheels *A* and *B*; but it ought not be greater than is just sufficient to prevent jamming, in order that the force with which *C* must be pressed against *A* and *B* may not become unnecessarily great.

457. The angle of repose is the angle whose tangent is the coefficient of friction between the surfaces in contact and varies with the material. The construction for designing this disengagement is as follows. Construct an isosceles triangle *CDE*, with the angles at *D* and *E* each a little greater than the angle of repose, produce *CD* and *DE*, and lay off upon them, *DA* and *EB*, proportional to the radii of the wheels to be connected, and join *AB*. Then the proportion borne respectively by *AD*, *BE* and *CD*, to *AB* will be the proportion borne by the radii of the wheels *A*, *B* and *C*, to the line of centres.

458. Class III comprises those disengagements and reversing gear that act by sliding contact. A pair of toothed wheels of any kind may be thrown in or out of gear by sliding one of them along its axis until contact is broken. This disengagement belongs to the class in which motion is transmitted by pressure, so that the velocity ratio and the relative position of the pieces are definite, and the communication of motion abrupt, with the possibility of some shock or strain.

459. Another plan is to have the wheels always in gear, one of them being loose on the shaft, and giving motion to the shaft by means of a cone or its equivalent on the shaft that carries it. The most common kind of reversing gear which act by means of toothed wheels is shown in Fig. 6, Plate XXV. *A* is the driving shaft carrying a bevel wheel which drives in contrary directions a pair of bevel wheels *B* and *C*, that turn loose on the shaft to be driven, *DD*. A double clutch *E*, sliding on a key on the latter shaft and turning with it, is made by means of a loose collar and forked lever *L* to clutch the one or the other of the bevel wheels *B* or *C*, according to the direction in which the shaft *DD* is to rotate. Friction cones or cylinders, or toothed discs are used as clutches for this purpose.

460. Class IV refers to disengagements and reversing gear by belts. When rotation is transmitted from one shaft to another by means of a belt and a pair of pulleys the disengaging gear used is the fast and loose pulley. The belt is moved from one to the other by a lever called a belt-shifter. Reversing gear by means of belts with fast and loose pulleys can be arranged as follows: On the driven shaft is one fast pulley between two loose pulleys, two belts run to the driving shaft, one open, one crossed, so that the driven shaft can be made to rotate at will in either direction, by shifting either belt to the fast pulley; the driver carries a broad pulley. Or we may have two loose pulleys on the driven shaft, one carrying a crossed and one an open belt, and a double friction cone between the loose pulleys, sliding on the shaft and turning with it, by means of spline and key, so that the velocity and direction of rotation of either pulley can be imparted at will to the driven shaft. In this case the driving shaft has a pulley for each of the loose pulleys on the driven shaft; and a different velocity ratio for the two shafts, when rotating in opposite directions, can be obtained by varying the dimensions of the connected pulleys to suit the demands of the case. This suggests another class of motions called quick return motions. A quick return motion is a combination in reversing gear, whereby the return path of a reciprocating working piece is travelled more quickly than the path during which useful work is done. Its object is economy of time.

461. PLATE XXV shows some of the simpler forms of disengaging and reversing gear. FIG. 1 represents a belt shifter for two belts, *a* and *b* are the forks that move the belts on and off their respective pulleys. *AB* is the line of ceiling, *H* and *H* are the hangers, the rod *KK* is capable of a longitudinal motion, through slots in *H* and *H* and is moved by the lever *SS*, which has its centre of motion at *C* and is attached to the sliding bar at *F*. Inspection of FIG. 15 will show an application of this belt shifter. The two shafts *PP'* and *HL* are in the same horizontal plane, and are connected by two belts. *A* and *B* are idle pulleys and *C* is a fast pulley. A belt shifter, of the form shown in FIG. 1, being adjusted between the two shafts, so that one belt runs in the fork *a* and the other in the fork *b*, will, by means of the lever, permit the shifting of the two belts from *C* and *B* (FIG. 15) to *A* and *C* and *vice versa*. These forks which act as guides must always enclose the advancing part of the belt.

462. FIG. 2 represents a coupling clutch of the 1st class. *AB* is the line of ceiling, *H* and *H'* are the hangers, and *KK* is the line of shafting. *P* is a loose pulley driven by a belt from the source of power; *D* and *F* are the two parts of the clutch; *F* is fast to and part of the pulley *P*, and *D* is keyed to the splined shaft *KK* and turns with it, but is capable of a longitudinal motion in the direction of that shaft by means of the lever *SS* pivoted at *C*. *P* being loose on the shaft *KK*, does not impart its motion of rotation to that shaft, but if we move the part of the clutch *D*, so that the pins fit the recesses in *F*, then *F* and *D* will move as one piece, taking the motion of *P*, which is fast to *F*, and the shaft *KK* will take up the motion of rotation of *P*.

P may be a fast pulley, and the shaft may be a discontinuous one broken between *D* and *F*, in this case the action of the clutch will be to make the entire shaft turn as one piece, by the engagement of *D* and *F*.

463. FIG. 3 shows another form of coupling clutch, that is of frequent use. The drawing explains itself. The object is the same as that described in FIG. 2, and similar parts are similarly lettered. FIG. 3*a* is another form of clutch, showing cross section of driver and follower and front elevation; the projections *b* fitting into the recesses *a*, and the two parts there turning

as one. The arrangement of the lever that moves the driver on its key along the shaft is similar in all these clutches; the centre of motion may be either above or below the shaft that carries the clutch.

464. FIG. 4 is a pair of friction cones for the same purpose as 2 and 3. *AB* is the ceiling, *H* and *H* are the hangers, and *KK* is the shaft. *P* is a loose pulley, connected to its driver by a belt; *F* is one of the cones, and is fast to, and part of *P*, its inner rim is turned accurately to a true conical surface as indicated; *D* is the other cone with its rim turned to a true conical surface, concentric with that of *F; D* is keyed to the shaft and turns with it, but is capable of a longitudinal motion in the direction of the axis of the shaft, by means of the lever *S*, with its centre of motion at *C*. This lever, with a collar at *L*, moves the cone *D*, and its motion causes *D* and *F* to unite as one piece, the pressure on, and consequent friction of the two concentric conical surfaces of *D* and *F*, cause *D*, and consequently the shaft, to take up the rotation of *F* and *P*.

465. FIG. 5 is a form of cylindrical friction clutch, *PP'* is one of the cylinders, with its inner rim turned true; suppose this to be attached to a loose pulley, that is driven by a belt from the driver; *LL'* is the section of the concentric cylindrical sectors, that can be brought in contact with the inner rim of *PP'*, whereby the friction between the cylindrical surfaces causes them to turn as one piece. *LL'* is fast to the shaft and turns with it. *bE* and *bE* are short levers, centred at *b*, with adjusting screws *II'*. When the point *E* is raised, the sectors are moved from the axis, when the points *EE* move towards the shaft, a spring causes the contact between the two concentric cylindrical surfaces to be broken. This raising the points *E* and *E'*, is accomplished by means of a loose collar, with conical point, and a forked lever *S*, pivoted at *O*. The point of this collar being forked between the points *E* and *E'*, by means of the lever *S*, causes them to lift, and by reason of their leverage, about *b* and *b*, to accomplish a close contact between *PP'* and *LL'*, whereby the rotation of *PP'* is communicated to *LL'*, and consequently to *KK*. The screws *I* and *I'* can be so adjusted, as to make a greater or less movement of *S* accomplish the desired result. A spring, not shown,

constantly urges *L* and *L'* towards the axis of the shaft.

This is a modern type of friction clutch, and is largely used on machine tools of medium size. These four types illustrate the general types of clutches used for disengaging alone, and variations of these can be applied to almost any case.

466. FIG. 6 shows a very common type of reversing gear. *A*, *B* and *C* are three equal bevel wheels, in gear. *A* is fast to its shaft and is the driver, *B* and *C* are loose on *DD;* *L* is a forked lever, centered at *O*, by which the clutch *E* can be moved in either direction longitudinally, *E* is the clutch, keyed to the shaft and turning with it, but capable of longitudinal motion in either direction, parallel to the axis of the shaft. In the bevel train *ABC*, *B* and *C* turn in opposite directions, so that the shaft *DD* can be made to remain at rest, or rotate in either direction, by causing the clutch *E* to remain in the position now shown, or to connect with either *B* or *C*, as the parts of the clutch *FF* are part of *B* and *C* respectively. If *DD* be the driver, *A* can be driven in either direction, or be made to remain at rest by the same means. The clutch *EE* is free to turn with the shaft and turns freely in the collar formed at the end of the lever *L*. This is both a disengaging and reversing gear.

467. FIG. 7 shows a disengaging and reversing gear with a quick return motion. That is, the driven shaft can be made to rotate more rapidly in one direction than in the other. This is true whichever shaft is the driver. The relation of the parts is the same as in FIG. 6.

468. FIG 8 shows a method of connecting two parallel shafts *AA'* and *BB'* by means of a clutch *C*, pulleys, and two belts. *AA'* is the driver; *E* and *E'* are two loose pulleys on *BB'*, *E* carrying an open belt, and *E'* a crossed belt; *C* is the coupling clutch, keyed to *BB'* and turning with it, but capable of a longitudinal motion in the direction of the axis of *BB'*, and turning freely, in the collar or fork at the end of the lever *S*, centred at *O*, that gives it its motion. If *AA'* turn right-handed, *BB'* will turn right handed or left-handed according as the clutch is moved to connect with *E* or *E'*. This is a disengaging and reversing gear.

If *BB'* is the driver and turns right-handed, *AA'* will turn right-handed or left-handed, according as *C* is fast to *E* or *E'*.

469. FIG. 9 and FIG. 9*a* illustrate the manner of disengagement and engagement by rolling contact. *C* is an eccentric axle, and when *H* is turned 90° to the position in 9*a*, *C* is raised and there is no longer rolling contact between *A* and *B*. *A* is the driver and can only drive *B* by rolling contact, when *C* is in contact with *A* and *B* as in FIG. 9.

470. FIG. 10 and 10*a* is a reversing motion. The idle wheels *B* and *C* cause *A* and *D* to turn in the same or in different directions, according as there are 3 or 4 wheels in the train. *F* is a movable frame, with its centre of motion at *K*, the axis of the driver *A;* motion, sufficient to break, or make connections, between *D* and *B*, and *D* and *C* is possible by means of the handle *H*. This reversing gear, composed of a movable frame and idle wheels, is of common use in a great many machine-tools, and is susceptible of many variations. The principle in all is the same.

471. FIG. 11 shows a reversing gear with a quick return motion. It is desired to impart to *TT*, a motion of rotation, which shall be in either direction at will, and the angular velocity of *TT* shall be less when turning in the same direction as, than when turning in the opposite direction from *SS*. Then $\frac{E}{D}$ must be greater than $\frac{A}{C}$. *B* is an idle wheel, for reversing the direction of rotation of *TT*. The shaft *SS* carries *E*, a tight pulley, the piece *AA'F*, loose on the shaft, the idle wheel *I* and the tight pulley *H;* a shifting belt, with shifter, connects with the source of power. When the belt is on *H*, the shaft *SS* and the pulleys *H* and *E* turn in one direction, and the pulley *D* and the shaft *TT* turn in the opposite direction, with a velocity ratio equal to $\frac{E}{D}$; when the belt is shifted to *F*, the piece *FA'A* turns in the same direction as *H* in the previous case, but the shaft *TT*, owing to the idle wheel *B*, on the intermediate axis *VV*, turns in the same direction now as *SS*, but in the opposite direction, from that in which it turned when the belt was on *H*, and with a velocity ratio equal to $\frac{A}{C}$ which is less than $\frac{E}{D}$. *A'* is a pipe,

loose on *SS*, that connects *A* and *F*, making them one piece.

472. FIG. 12 is an example of disengaging and reversing gear acting by sliding contact. *SS* is a shaft, capable of a sliding motion in bearings in the direction of its axis, *E* is the driver, and carries the bevel wheel *B* which is keyed to and turns with it. *SS* can then be made to rotate in either direction, by moving it longitudinally, so that *B* is in gear with either *A* or *C*. Then if *SS* rotates right-handed, when *B* gears with *C* as in the figure, it will rotate left-handed, if *SS* be moved until *A* gears with *C*.

If *SS* be the driver, the shaft *E* can be driven right or left-handed, or can be made to remain at rest according as *B* gears with *A*, *C*, or neither. An arrangement similar to that in FIG. 7 will give a quick return motion. That is, in FIG. 12, *A* may be larger than *C*, and a larger wheel may be keyed to *E* above *B*

473. FIG. 13 is a variation of FIG. 6. The object is to drive the shaft *D* in either direction, by means of the bevel train, composed of three equal wheels *A*, *B* and *C*, *A* and *C* being loose pulleys, and the clutch *N*. *I* is an idle wheel, *F* is the fast pulley, connected by a shifting belt with the source of power. The rotation of *F* is communicated to the shaft *EE*, and thence right-handed or left-handed to *D*, according as the clutch *N* is made to connect with *A* or *C*. The shaft *EE* is supposed to be turning right-handed, as viewed from the end farthest from the driving pulley *F*. The details of the coupling clutch, lever and beltshifter, or belt shipper, are identical with those in the similar mechanisms already explained. If the belt be shifted to *I*, the shaft *EE* ceases to rotate If the belt be on *F* the shaft *EE* rotates, but if the clutch *N* be situated as in the drawing, the shaft *D* will receive no motion of rotation from *EE*, as *A* and *C* are loose pulleys, and not in contact with *N*.

474. FIG. 14 shows a disengaging and reversing gear; the object being to communicate to *TT* a motion of rotation in either direction.

It consists of a shaft *SS*, with *E*, a fast pulley, *I* an idle pulley, *F*, connected by the pipe *HH* to *A*, and forming one piece of *F*, *H* and *A*, this piece riding loose on the shaft *SS;* also the bevel train, composed of three equal wheels *A*, *B* and *C* of which *B* is fast to *TT* and turns with it. *A* is loose on *SS*, but fast to *H* and *F*, *C* is fast to *SS* and turns with *SS* and *E*. A shifting belt connects with the source of power and can be shifted, to rest on *E*, *I* or *F*.

When the belt is on *E*, *SS*, *E* and *C* turn in one direction, and *TT* turns in a certain direction; when the belt is on *I* the system is at rest; when the belt is on *F*, the piece *FHA* turns in the same direction as *SS* did when the belt was on *E*, but *TT* turns in a direction opposite to that first described, as do *SS*, *E* and *C*, and the desired result is obtained, that of giving motions of rotation to *TT* in either direction at will.

475. A quick return motion can be obtained by applying the principles suggested in the description of FIG. 7.

476. FIG. 13 and FIG. 14 represent mechanisms, that can only drive, as described. In FIG. 13 *EE* must be the driver and *D* the follower, and in FIG. 14 *TT* is the follower.

477. FIG. 15 illustrates the manner of connecting two parallel shafts *PP'* and *HL* by two belts, one open, one crossed, and a belt shipper. Let *PP'* be the driver, carrying the fast broad pulley *V*; *HL* then is the follower and carries the two loose pulleys *A* and *B*, and the tight pulley *C*, and the two shafts *HL* and *PP'*, with the open belt on *C*, turn in the same direction with a velocity ratio of $\frac{V}{A}$. Suppose the belts to be shifted until the crossed belt is moved to *C* and the open belt to *A*; *A* being an idle pulley, the open belt has no effect on *HL*, but the crossed belt being on the tight pulley *C*, the two shafts *PP'* and *HL* will turn in opposite directions and with the same velocity ratio as before.

478. FIG. 16 shows a manner of connecting two shafts *QQ'* and *NM*, so as to give a quicker velocity of rotation in one direction than in the opposite direction, thus affording the possibility of a quick return motion. If the work is done when the open belt is on the tight pulley *D*, then the return stroke will be accomplished by shifting the belts until the crossed belt is on the tight pulley *K* and the open belt on its loose pulley *E*. The forward stroke has a velocity ratio of $\frac{S}{D}$, *QQ'* being the driver; the re-

turn stroke has a velocity ratio of $\frac{S}{K}$, K being less than D; the return stroke has the greater velocity in the ratio of $\frac{D}{K}$, and the combination is therefore one which gives a reversing and quick return motion. It is used largely for machine tools, where the desire exists to accomplish as far as may be, the greatest saving of time. Fig. 1 is a form of belt shipper that could be adapted to the use mentioned in Figs. 15 and 16.

PLATE XXVI.

Hydraulic Connections.

479. An elementary combination in mechanism consists of a pair of primary moving pieces so connected that one transmits motion to the other; the kinds of motion of the two pieces are determined by the manner of connection; in general this connection is effected by actual contact, by bands or belts, and by links, arms or levers; in addition to these connectors, a confined mass of fluid can be used as connector, in an elementary mechanical combination.

480. A combination, with hydraulic connectors, is one in which two cylinders, fitted with movable pistons, are connected with each other by a passage, and the space between the pistons is entirely filled with a mass of fluid of invariable volume.

481. Any liquid mass may be treated, in practical questions, concerning the transmission of motion, as if its volume were constant. If water be confined in a perfectly rigid, prismatic vessel, and a pressure of one pound per square inch be applied to the surface of the liquid, the compression in the direction of the depth of the vessel, would be $\frac{1}{300000}$ of the depth of the liquid.

482. Each particle of a liquid at rest, must exert and receive equal pressures, to and from all directions. If this were not so, the particles of a liquid, agitated by an extraneous force, could not come to a state of rest. The principle of the equal transmission of fluid pressure asserts, 1st, that equal surfaces of the sides of a vessel, containing a liquid, receive equal pressures, at equal depths below the surface; 2nd, if a closed vessel be filled with a liquid, assumed to have no weight, and an aperture of one square inch area, be made in one side of it, and fitted with a piston, on which a pressure of ten pounds is exerted, that there will be a pressure of ten pounds exerted on every square inch of the surface of the containing vessel; consequently, if another aperture, of 100 square inches area, be made in the side of the vessel, and a cylinder and piston be fitted therein, that this

piston will receive a pressure of 1000 pounds, when there is a pressure of ten pounds exerted on the piston of one square inch area. Upon this principle, and the assumption of the incompressibility of fluids, the hydraulic press and hydraulic connections generally depend.

483. The volume of the mass of fluid confined between two pistons, being invariable, it follows that, if one piston, the driver, moves inward, sweeping through a certain volume, the other piston, the follower, must move outward, sweeping through an equal volume ; for these two pistons, with the connecting mass of fluid, of invariable volume, constitute an elementary combination in mechanism.

484. A piston is a primary piece, sliding in a vessel, called a cylinder. Ordinarily, the piston path is rectilinear, hence the bearing surfaces of the cylinder and piston are ordinarily cylindrical, see Fig. 11, Plate XXVI. When the motion of a piston is circular the bearing surfaces of the cylinder and piston are surfaces of revolution.

485. A plunger differs from a piston, in that, while the bearing surfaces of cylinder and piston are in sliding contact, the plunger has no bearing surface with the cylinder proper, but with a collar inserted in the cylinder and of a depth sufficient to prevent the escape of the confined fluid. See Fig. 9, Plate XXVI. The bearing surface of the plunger, is equal in length, to the thickness of the collar plus the length of the stroke. The piston has a bearing surface, of a depth, sufficient to prevent the escape of the fluid, but in this case the cylinder has a bearing surface equal, in length, to the bearing surface of the piston plus the stroke.

486. The action of plungers and pistons, in transmitting motion is exactly the same, and the word piston is used to include plungers and pistons.

487. The volume, traversed by a piston in a given time, is the product of two factors, the transverse area of the piston and the stroke of the piston. The transverse area is that of a plane, normal to the line of direction of the stroke, and bounded by the bearing surface of the piston and cylinder. In a rectilinear sliding piston, that plane is normal to the axis of the cylinder.

In a curvilinear moving piston, the area is that of a projection of the piston on a plane normal to its direction of motion. When the motion of the piston is rectilinear, the length of the volume traversed, is the distance moved by each point of the piston. When the motion is curvilinear, that length is the distance traversed by the centre of area of the piston.

488. So long as the transverse area and length of the space traversed by the piston are the same, the form of the ends of the cylinder does not effect the volume of the space traversed. When the space in the cylinder, which contains the fluid acted on by a piston, is traversed by a piston rod, the effective transverse area is equal to the transverse area of the piston, diminished by the transverse area of the piston rod. In that division of the cylinder, not traversed by the piston rod, the entire transverse area of the piston is effective.

489. From the equality of the volumes, traversed by a pair of pistons in hydraulic connection with each other, it follows that the velocities of the pistons are inversely as their transverse areas, measured on planes normal to the directions of their motions.

490. It may be required to find the comparative velocities, with which the fluid particles flow through a given section of the passage which connects a pair of pistons. The mean velocity of flow through a given section of the passage, is the mean value of the component velocities of all the particles that pass through it, in a direction normal to that section; from the fact that, in a given time, equal volumes of fluid flow through all sectional surfaces that extend completely across the passage, it follows that the mean velocity of flow through any such section, is inversely as its area, and this principle applies to all possible sections, transverse, oblique, plane or curved.

491. Valves are used to regulate the communication of motion, with hydraulic connectors, by opening and closing passages through which the fluid flows. For example, a cylinder may be provided with valves which shall cause the fluid to flow in through one passage, and out through another. Gf this use of valves, two cases may arise. First, when the piston drives the fluid, the valves may be self-acting, that is, moved by the fluid.

If there be two passages into the piston, one provided with a valve opening inward, and the other provided with a valve opening outward, then, during the outward stroke of the piston, the former, or inward opening valve, is opened, and the latter, or outward opening valve is closed, by the inward flow of the liquid through the inward opening valve, following the motion of the piston ; during the inward stroke of the piston, the former, or supply valve is closed, and the latter, or outlet valve, is opened by the pressure of the liquid, and through it, the liquid is forced out of the cylinder. This combination of cylinder, piston and valves constitutes a pump. Second, when the fluid drives the piston, the valves must be opened and closed by mechanism, or by hand. In this case the cylinder is a working cylinder, as in the case of an engine run by high pressure water. See article 270 Goodeve's principles of mechanics.

492. It is by the aid of valves that intermittent hydraulic connection between two pistons is effected ; and the action produced is analogous to that of the click, ratchet, and detent, in intermittent linkwork., For example, in the hydraulic press, the rapid motion of a small plunger, in a pump, causes the slow motion of a large plunger in a working cylinder, and the connection of the two plungers is made intermittent by means of the discharge valve of the pump, which is a valve, opening outwards from the pump cylinder and inwards to the working cylinder.

The pump draws water from a reservoir, and forces it into the working cylinder, during the in-stroke of the pump plunger, the plunger of the working cylinder moves outward with a velocity as much less than that of the pump plunger, as its area is greater; at the end of the in-stroke of the pump plunger, the valve between the working cylinder and the pump closes, and prevents any back flow to the pump, and it thus answers the purpose of the detent in ratchet work. During the out-stroke of the pump plunger, this valve remains closed, being an outward opening valve for the pump cylinder, and the plunger in the working cylinder remains at rest, while the pump is filling its cylinder with water, through an inward opening valve. When the plunger of the working cylinder has completed its out-stroke, which may be of any length, and may require any number of strokes of the pump plunger, it can be moved inwards by opening a valve by hand, provided for the purpose, and permitting the water to escape; a hydraulic press is sometimes provided with more than one pump, thus producing a continuous outward stroke of the plunger in the working cylinder, as the pumps work in succession, and not simultaneously; this is analogous to the action of the double acting click, FIG. 13, PLATE XVI.

✓ 493. VALVES IN GENERAL, considered with reference to the means by which they are moved, may be divided into three principal classes: 1st, Valves, sometimes called *clacks*, which are opened and shut by the pressure of the fluid that traverses their openings, and are usually intended for the purpose of permitting the passage of the fluid in one direction only, and stopping its return; 2nd, valves moved by hand; 3rd, valves moved by mechanism. When a piston drives a fluid, as in ordinary pumps, the valves are usually moved by the fluid: when the fluid drives the piston, it is in general necessary that the valves should be moved by hand or by mechanism. In water pressure engines that work occasionally and at irregular intervals, such as hydraulic hoists and cranes, the valves are usually opened and shut by hand; in those which work periodically and continuously, they are moved by mechanism connected with the engine.

Safety valves for permitting a fluid to escape from a vessel when the pressure tends to rise above the limit of safety, belong to the class that are moved by the fluid. Regulating valves are adjusted either by hand, or by means of a governor.

The SEAT of a valve is the fixed surface on which it rests, or against which it presses.

The FACE of a valve is that part of its surface which comes in contact with the seat.

When a valve occurs *in the course* of a pipe or passage, the valve box or chamber, being that part of the passage in which the valve works, should always be of such a shape as to allow a free passage for the fluid when the valve is open, so that the fluid may pass the valve with as little contraction of the stream as possible; and if necessary for that purpose, the valve chamber may be made of larger diameter than the rest of the passage.

The usual materials for valves and their seats are iron, bronze, brass, hardwood, leather, india rubber, and gutta percha.

When a valve and its seat are both of metal, they should be of the same metal; for when they are of different metals, a galvanic action takes place, which causes one or the other of them to be corroded.

In water pressure engines and pumps, the best material for the seats of metal valves is some hard wood, such as elm or lignum vitæ, the fibres being set endways, and constantly wet.

India rubber and gutta percha being dissolved or softened by oils, whether fatty or bituminous, are unsuitable materials for valves to which those fluids have access.

494. THE BONNET OR CONICAL VALVE is a flat or slightly arched circular plate of metal, whose face, *ab*, being formed by its rim, see Fig. 1, Plate XXVI, is sometimes a frustum of a cone, and sometimes the zone of a sphere, the latter figure being the best. Its *seat cd*, being the rim of the circular orifice which the valve closes, is of the same figure with the face or rim of the valve, and the valve face and its seat are turned and ground to fit each other exactly, so that when the valve is closed no fluid can pass. The thickness of a valve of this form is usually from a fifth to a tenth of its diameter, and the mean inclination of its rim about 45°.

To insure that the valve shall rise and fall vertically and always return to its seat in closing, it is sometimes provided with a *spindle s*, as shown in Fig. 1, being a slender round rod perpendicular to the valve at its centre, and moving through a ring or cylindrical socket *f*. A knob *k* on the end of the spindle prevents the valve from rising too high. When the valve is to be moved by hand or by mechanism the spindle may be continued through a stuffing box, and connected with a handle or a lever, so as to be the means of transmitting motion to the valve.

When the valve seat is at the upper end of a cylindrical passage, as in ordinary safety valves, in place of the spindle a tail is often supplied.

Fig. 2 is an elevation of the valve, showing the *tail*, by which it is guided so as to move vertically, and to return always to its seat. Fig. 3 is a horizontal section of the tail, which consists of three vertical ribs or "feathers," radiating at angles of 120°. Their outer surfaces or edges are small portions of a vertical cylinder, turned to fit the cylindrical tube on which the valve is placed easily, but not too loosely.

495. THE COMMON SAFETY VALVE used for steam boilers as well as for water pressure engines, is a bonnet valve loaded with a weight equal to the greatest excess of the pressure upon each area equal to that of the valve within the vessel on which the valve is fitted, above the pressure of the atmosphere, to which it is safe to subject that vessel during its ordinary use.

Sometimes the valve has a vertical spindle rising from it, moving in guides, and loaded directly with cylindrical weights which rest on a collar that surrounds the spindle.

Sometimes the load is applied by means of a lever, as in FIG. 4, which represents a section of the valve seat and valve, and an elevation of the lever. *A* is the valve, *D* a stud or knob in the centre of its upper side, *CB* a lever jointed to a fixed fulcrum at *C*, *W* the weight, which can be shifted to different positions on the lever, so as to vary the load on the valve.

✓496. THE BALL CLACK (FIG. 5) is a valve of the form of an accurately turned sphere. When of large size, it is in general hollow, in order to reduce its weight. Its face is its entire surface: its seat is a spherical zone, as in the case of some bonnet valves already referred to. As the ball clack *B* fits its seat alike in every position, it needs neither spindle nor tail; but either the chamber in which it works must be of such a shape and size as to insure its always falling into its seat, or the same object must be effected by means of wire guards *ff* enclosing it, as shown in the figure. The latter plan is the better, as it is the more likely to insure that there shall always be a free passage for the fluid round the valve when open.

497. DIVIDED CONICAL VALVE. Bonnet valves of large size, when working under high pressures, often require an inconveniently great amount of work to open them, and shut with such violence as to cause injurious shocks to the machine. To obviate this evil, a valve has sometimes been used, composed of a series of concentric rings. The largest ring may be considered as a bonnet valve, in which there is a circular orifice, forming a seat

for a smaller bonnet valve, in which there is a smaller circular orifice, forming a seat for a still smaller bonnet valve, and so on. This arrangement enables a large opening for the passage of water to be formed with a moderate upward motion of each division of the valve ; and consequently with a moderate expenditure of work to open it, and a moderate shock when it shuts.

498. THE DOUBLE-BEAT VALVE is the best contrivance yet known for enabling a large passage for a fluid to be opened and shut easily under a high pressure. FIG. 6 represents a section of the valve, with its seats and chamber, and FIG. 7 a plan of the valve alone, on a reduced scale.

The valve shown in the figure is for the purpose of opening and shutting the communication between the pipes *A* and *B*.

The pipe *B* is vertical, and its upper rim carries one of the two valve seats, which are of the form of the frustum of a cone, and each marked *a'*.

A frame *C*, composed of radiating partitions, fixed to and resting on the upper end of the pipe *B*, carries a fixed circular disc, *ss* whose rim *aa* forms the other conical valve seat.

The valve *D* is of the form of a turban, and has two annular conical faces, *ff* which, when it is shut, rest at once on and fit equally close to the two seats *a'*, *a'*, and *a*, *a*. When the valve is raised, the fluid passes at once through the cylindrical opening between the lower edge of the valve and the upper edge of the pipe *B*, and through the similar opening between the upper edge of the valve and the rim of the circular disc.

The greatest possible opening of the valve is when its lower edge is midway between the disc and the rim of the pipe *B*.

If the upper and lower seats are of equal diameter, the valve is little affected by any excess of pressure either in *A* or in *B*; and a force a little exceeding its own weight is sufficient to open it. It is then called an *equilibrium valve.*

If the diameter of the upper seat is the less, an excess of pressure in *A* over *B* tends to keep it shut, and an excess of pressure in *B* over *A* to open it.

If the diameter of the upper seat is the greater, an excess of pressure in *A* over *B* tends to open the valve, and an excess of pressure in *B* over *A* to keep it shut. This arrangement is seldom used.

In each case, the force arising from difference of intensity of pressure, and tending to open or shut the valve, as the case may be, is nearly equal to that difference multiplied by the difference between the area of the pipe *B* and that of the circular disc.

The equilibrium valve is the kind of double-beat valve most commonly used in steam engines. In water pressure engines, pumps, and hydraulic apparatus generally, the lower valve seat is generally made a little larger than the upper.

✓499. A FLAP VALVE, illustrated by FIG. 8, is a lid *a* which opens and shuts by turning on a hinge, *h*. The hinge may either be a metal joint, or may be provided by the flexibility of the material of the valve itself, when that is leather or india rubber.

The face may be of leather, india rubber, or metal; in the last case the face and seat should be carefully scraped to true planes.

In hydraulic machines, the most common material for flap valves is leather, which should, as far as possible, be kept constantly wet. A large leather flap may be stiffened in the middle by a plate of wood or metal.

A pair of flap valves placed hinge to hinge (usually made of one piece of leather fastened down in the middle) constitute a "*butterfly clack.*" The chamber of a flap valve should be of considerably greater diameter than the valve.

500. THE DISC AND PIVOT VALVE, OR THROTTLE VALVE, consists of a thin flat metal plate or disc, which, when shut, fits closely the opening of a pipe or passage, generally circular in section, but sometimes rectangular. The valve turns upon two pivots or journals, placed at the extremities of a diameter traversing its centre of gravity, so that the pressure of the fluid against it is balanced about its axis of rotation. and the valve can be turned into any angular position by a force sufficient to overcome its friction.

The *face* of this valve is its rim; its *seat* is that part of the internal surface of the passage which the rim touches when the valve is shut; and those surfaces ought to be made to fit very accurately, without being so tight as to cause any difficulty in opening the valve.

One of the journals of the valve usually passes through a bush or a stuffing box in the pipe, so as to afford the means of communicating motion to the valve from the outside.

It is difficult to make valves of this class perfectly water-tight or steam-tight without too much impeding their motion. They are, therefore, not so well suited for stop valves as for regulating valves, and for the latter purpose they are much used, both in water pressure engines and in steam engines.

501. SLIDE VALVES, FIG. 12. The *seat s* of a slide valve consists of a plane metal surface, very accurately formed, part of which is a rim surrounding the orifice or *port*, *P*, which the valve is to close, and from $\frac{1}{4}$ to $\frac{1}{20}$ of the breadth of that orifice, while the remainder extends to a distance from the orifice equal to the diameter of the valve, in order that the valve, when in such a position as to leave the port completety open, shall still have every part of its face in contact with the seat.

The valve *V* is of such dimensions as to cover the port together with that portion of the seat which forms a rim surrounding the port. The face of the valve must be a true plane, so as to slide smoothly on the seat, and, in large slide valves, consists of a rim surrounding that central part of the valve which directly closes the orifice, and which is more or less concave, to enable it the better to resist the pressure which acts on the back of the valve when it is closed.

Very large slide valves, such as those in the course of the main water pipes of large towns, are strengthened at the back by flanges or ribs.

The valve and its seat are contained within an oblong box or case, *B* large enough to permit the easy motion of the valve within it, and usually forming an enlargement in the course of a pipe. The *valve rod*, *R*, by means of which the valve is opened and shut, passes out through a stuffing box.

502. A PISTON VALVE is a piston *P* moving to and fro in a cylinder, *C*, whose internal surface is the *valve seat*, FIG. 13. The *port F* is formed by a ring or zone of openings in the cylinder, communicating with a passage which surrounds it; and by moving

the piston to either side of these openings, that passage is put in communication with the opposite end of the valve cylinder.

503. COCKS. This term is sometimes applied to all valves which are opened and shut by hand, but its proper application is to those valves which are of the form of a frustum of a cone, or conoid, turning in a seat of the same figure.

In the most common form of cock, the seat is a hollow cone of slight taper, having its axis at right angles to the pipe in whose course it occurs. The valve is a cone fitting the seat accurately, and having a transverse passage through it of the same figure and size with the bore of the pipe, so that in one position it forms simply a continuation of the pipe, and offers no obstruction to the current, while by turning it into different angular positions, the opening may be closed either partially or wholly. A screw and washer at the smaller end of the cock serve to tighten it in its seat.

504. A PLUNGER is a metal cylinder, closed at the ends, and accurately turned on the cylindrical surface, which, in a single acting pump or water pressure engine, acts at once as piston and as piston rod, by having a reciprocating motion in a cylinder. The internal diameter of the cylinder is larger than that of the plunger by an amount sufficient to prevent their touching. Round the circular aperture through which the plunger works is a water-tight "cupped leather collar." A section of a cylinder showing a plunger working in it is given in FIG. 9.

The area of the transverse section of the *plunger*, and not that of the cylinder in which it works, is to be used in computing the effort exerted by the pressure of the water upon it.

The weight of a plunger is often made considerable, and sometimes a load also is placed upon it, in order that energy may be stored in lifting it, and restored when it descends, as in the accummulator.

505. THE CUPPED LEATHER COLLAR through which a plunger works is shown in section on a small scale at *Q* in FIG. 9, and on a larger scale in FIG. 10. It resembles in shape an inverted annular channel; and is lodged in an annular recess surrounding the plunger. Its hollow channel is turned towards the bottom of the cylinder; and the water, tending to enlarge that channel,

presses its outer side against the recess, and its inner side against the plunger, and so keeps a water-tight joint.

The friction between a plunger and its leather collar is given approximately by the following formula: let d be the diameter of the plunger, in inches; p, the pressure, in lbs. on the square inch; R', the friction, in lbs., then

$$R' = f p d.$$

According to experiments, $f =$ about $1.2 \times$ the depth of bearing surface of the collar; and the friction is, roughly, one-tenth of the load in ordinary cases; according to other experiments, f ranges from .05 to .03 of the load.

506. LEATHER PACKED PISTON. A piston is distinguished from a plunger by accurately fitting the cylinder in which it works, so as to be water-tight, and by being of no greater thickness than is necessary to make it water-tight. It is attached to a rod, strong enough to transmit the effort that acts on it to the mechanism which it drives. The water acts on one face of the piston, or on both, according as the engine is single acting or double acting. See FIG. 11.

When the water acts on that side of the piston from which the rod extends, the cylinder cover has a stuffing box in its centre, through which the rod works; and the opening is made water-tight by a leather collar, as already described, or by hempen packing.

When the piston is to be packed by means of leather, its disc, which fits the cylinder easily (and to which the rod is firmly attached by a screw, or a screw and nut, or a key), is made slightly concave on the upper and under faces; then on each of those faces is placed a leather ring, shaped somewhat like a saucer with a hole in the centre, and having its edge turned up all round so as to press flat against the inside of the cylinder for a breadth of an inch, or an inch and a half, or thereabouts. The edges of those leather rings are thus turned opposite ways, that of the upper ring upwards, and that of the lower ring downwards. Each of the rings is held in its place by a round saucer-shaped guard or piston cover, bolted or screwed to the body of the piston.

The friction of such pistons, like that of plungers, is found to be about *one-tenth* of the effort of the water.

A piston, like a plunger, may be loaded for the purpose of storing energy, and according to the same principles.

507. HEMPEN PACKING. The body of a piston which is to be packed with hemp is from two to four inches less in diameter than the cylinder in which it is to work; and its depth is about one-sixth of the diameter of the cylinder. It bulges a little at the middle of its depth. Round its base there projects a horizontal flange, whose rim fits the cylinder easily. Above that flange and round the body of the piston is wrapped the packing, consisting either of loose hemp, or of a soft loosely spun hempen rope, called a "gasket," soaked with grease. Above the packing is a ring of the same size and figure with the flange, for pressing the packing down, and causing it to fit tightly in the cylinder. This "junk-ring" is held down and can be moved towards the flange so as to compress the packing when required, by means of screws.

The stuffing box of a piston rod is packed with hemp in a similar manner, the hemp being pressed down and made to fit tightly round the piston rod by means of the stuffing box cover and its bolts or screws.

508. THE HYDRAULIC PRESS is supplied with water from an artificial source, and is therefore not a prime mover, but a piece of mechanism for conveniently applying the energy of the muscular power, or steam power, by which its supply pumps are worked. It exemplifies in a simple form various parts which enter into water pressure engines generally.

FIG. 9 is an elevation of a hydraulic press supplied by a hand forcing pump, and a vertical section of the cylinder and pump; and FIG. 10 represents the plunger collar.

A is the press cylinder, made thick enough to resist the pressure. The bottom should be segmental or hemispherical, not flat. *B* is the plunger; *Q* its collar; *C* a plate carried on the head of the plunger; *D* the upper plate of the press; *E* standards guiding the motion of the plate *C*, and strong enough to resist a working tension equal to the force to be exerted by the plunger. *F* is the pump cylinder, *I* its plunger, and *K* a guide for the plunger rod. *G* is the pump handle; *H* and *H'* are alternate centres, about either of which it can be made to work, so as to give a greater or a less leverage as required. *L* is the supply pipe of the press cylinder, through which water is forced into it by the pump. It contains a self-acting clack, *N*, opening towards the press cylinder, to prevent the return of water towards the pump. *M* is the supply valve or suction valve of the pump, being a self-acting clack opening upwards; *O* is the safety valve, *P* its weight; *R* the escape valve, or discharge valve, being a conical plug worked by means of a screw, kept shut while the plunger is being raised, and opened, so as to let the water escape from the press cylinder, when the plunger is to be allowed to descend by its weight. The discharge pipe, *S*, leading from this valve to a tank from which the pump draws its water, is the *tail race* of the machine.

The following method is given for computing the elements of the hydraulic press.

Let P, represent the applied power,
W, the resistance to be overcome,
L, the lever arm of the power,
l, the lever arm of the pump plunger,
A, the area of the plunger in the working cylinder.
a, the area of the pump plunger.
N, a factor, to provide for the allowance to be made for the friction in the pump, the friction of the plungers in their collars, and the resistances that occur, due to bends, contractions, and all impediments to perfect transmission; 1.3 is the value here allowed for N.

We have then the relation,

$$PLA = Wla \times N.$$

509. The hydraulic press can be worked with a head of water from natural sources, or from an elevated natural supply; the valve system is similar, permitting a downward and checking the backward flow. Water acts as a moving power, by its weight, by its *vis viva*, or by both.

The power of a fall of water is expressed by the equation $P = V w h$.

P being the power, V, the volume, w, the weight of a unit of volume, and h, the vertical height of fall; again $P = Vw\frac{v^2}{2g}$; when water acts by its weight and *vis viva* simultaneously, by combining the effect of an acquired velocity with the vertical fall, in acquiring that velocity, the effect is measured by the sum of the two effects in the two cases cited, and

$$P = \left(h + \frac{v^2}{2g}\right)Vw.$$

510. To compute the thickness of the pipe to resist a given pressure, we multiply the pressure, in pounds per square inch, by the diameter of the pipe in inches, and divide this product by twice the tensile strength of a square inch of the material, of which the pipe is constructed. Experiment has shown that a pipe 15 inches in diameter and ¾ of an inch thick, will support a head of water of 600 feet, and that a pipe of oak, 15 inches in diameter, and 2 inches thick will support a head of 180 feet.

Three thousand pounds is taken as the tensile strength per unit of area, of cast-iron in ordinary water pipes; this small value is taken on account of the liability of such castings to be of poor quality and imperfect from honeycombing and other flaws.

511. Historically, falling water was one of the earliest sources of power, but it is comparatively recently, that attention has been called to the advantage of using hydraulic power where there is no natural fall, for the purpose of driving certain classes of machines, especially those, whose use is intermittent. It has been found that high pressure water, of 700 pounds pressure per square inch, can be transmitted a mile with a frictional loss of only about two per cent., where the mains are properly proportioned and the head is constantly maintained. Steam pumps are used to pump the water into large cylinders provided with heavy plungers, weighted to produce the required pressure on the confined water; these are the accumulators; from these, pipes, provided with suitable valves, lead to the various points, where it is desired to apply power; at these points some suitable hydraulic motor is located, for the conversion of the power into the desired motion.

512. Machine tools for rivetting, punching and shearing iron plates, are very generally worked by water pressure. Heavy loads, such as parts of bridges, cars, and material in foundries are readily handled by hydraulic lifts and cranes.

Hydraulic power commends itself to the artillerist, on account of its simplicity, ease of transmission, adaptability to intermittent action, certainty of control from the slightest motion, to a blow requiring the entire power stored in the accumulator, ease of subdivision into small units at the required points of application, and from the fact that its use does not require belts, pulleys and shafting. Water at high pressure is less liable to freeze than at low pressure; the addition of one part of glycerine to four parts of water, prevents freezing as low as 16° Fahr.

513. Of late years the electrical connector has been extensively introduced. Its essential parts are a dynamo, a conductor and a motor, together with a belt or other means of connection between the dynamo and the source of power, and again between the motor and the machine at the desired point of application of power. The dynamo converts mechanical power into electrical power, the conductor transmits the electrical power to any desired point, where the motor transforms the power of the electrical current thus conveyed, into mechanical power, which is transmitted by the ordinary mechanical means to the machine to be worked. The dynamo and motor are essentially the same. If two belts form parts of the connection, six per cent. of the power, to be transmitted may be lost through the slip and rigidity of the belts; five per cent. may be lost in the dynamo's conversion of mechanical into electrical power and five per cent. may be lost in the motor.

The loss in the conductor will depend upon its length, material and cross section; but the total losses in electrical connection and transmission of power, will be small when compared with the losses due to the transmission of power by means of compressed air, steam, water, belting, or shafting, and especially so, if the power to be transmitted is large, or if the distance to which the power is to be transmitted exceeds six hundred feet, or if it is desired to subdivide the power, and intermittent action is desired, as a motor can be located at each point of application of power, as at the different machines or cranes in a shop, and they can be disconnected when not in use, while with shafting, belts and pulleys, at each point of application there must be at least, the continued rotation of the countershaft or idle pulley ; again the ability to cut out any unit, where subdivided power is used, permits a repair to be made, in case of accident, without stopping the entire system, then absolute regulation is more readily obtained with electrical transmission. In view of the cases cited, electrical connectors must be considered as admirably adapted for artillery purposes.

514. These notes have been prepared with the object of giving an outline of the different methods of transmitting power. These methods are, 1st mechanical transmission of power, by actual contact of two bodies, as in the use of friction cylinders and cones, and toothed wheels, connection by belts, shafting and pulleys, as in the ordinary machine shop, connection by links and levers, as the crank and connecting rod in the steam engine and the various kinds of linkage mechanisms. 2nd Hydraulic transmission of power, as in the hydraulic jack, the hydraulic press and accumulator, with the valves, and pipes to the different motors at the desired points of application of power, as machines for rivetting, punching and shearing. 3rd Electrical transmission of power, as in the general use of electrical motors to work cranes, and other machines.

PROBLEMS.

Assume as a source of power a horizontal revolving shaft, 3″ in diameter, and in every case draw a sketch or a diagram to scale where necessary.

Problem 1. Design a mechanism to give to a sliding piece a reciprocating straight line motion of four inches, the line of direction of the motion to pass through the centre of the shaft.

Problem 2. Design a mechanism to give a reciprocating straight line motion of four inches, the line of direction of this motion to pass three inches below the shaft.

Problem 3. Design a mechanism to give a reciprocating straight line motion of six inches to a piece, the line of direction of this motion to pass six inches below the centre of the shaft, and parallel to and twelve inches from the piece that connects with the shaft.

Problem 4. Design a cam for this shaft that will give a vertical straight line reciprocating motion of six inches, whose line of direction shall pass through the centre of the shaft.

Problem 5. Design a crank and connecting rod mechanism to give a reciprocating straight line motion of six inches, whose line of direction shall pass 1st through the centre, and 2nd above the centre of the shaft.

Problem 6. Design a mechanism of crank and infinite connecting rod, that shall give a reciprocating straight line motion of six inches.

Problem 7. Having given the mechanism that shall give to a sliding piece a reciprocating straight line motion of one foot, whose line of direction passes through the centre of the shaft, design a mechanism that shall derive a reciprocating straight line motion of three inches from the given mechanism, the motion to be the reverse of the former in its direction of reciprocation, and in a vertical plane parallel to the vertical plane containing the given motion, and above it.

Problem. 8. Design a mechanism that shall give to a reciprocating piece, a motion of ten inches in the vertical plane containing the axis of the shaft, the motion to be parallel in direction to said axis.

Problem 9. Design a linkage mechanism that shall give to a reciprocating piece a straight line motion of four inches, the period of advance to be to the period of return as three to two. Make a sketch showing a possible application.

Problem 10. Design a belt and pulley mechanism to give to a reciprocating piece a straight line motion, whose period of advance shall be to its period of return as two to one.

Problem 11. Design a mechanism consisting of belts, pulleys and gear wheels, that shall give to a reciprocating piece a straight line motion, the periods of advance and return to be as ten to one.

Problem 12. Design a mechanism wherein the straight line motion of a piece can be reversed at will, by means of an adjustable arm and idle wheel.

Problem 13. Design a reversing gear, using a Bevel train of wheels, showing more than one method, and a method of obtaining a quick return motion with bevel gear.

Problem 14. Show that a conical friction clutch and two wheels, one with open, one with crossed belt, can be made the equivalent of one belt, an idle and two tight pulleys.

Problem 15. Design a mechanism of rack and pinion and a reversing and quick return motion gear, and a simple reversing gear.

Problem 16. Design a mechanism that shall give a vertical reciprocating piece, a vertical straight line motion of eight inches, with periods of rest equal to one-half the periods of motion (Article 48, Goodeve).

Problem 17. How can periods of rest be obtained in the rack and pinion mechanism, or in any train of wheels for a stated period? Illustrate the process (Article 92, Goodeve).

Problem 18. Design a cam plate on a reciprocating piece, that shall give a reciprocating straight line motion of two inches in a direction at right angles to the first piece (Article 67, Goodeve).

Problem 19. Design a cam plate that shall give a reciprocating piece, a straight line vertical motion derived from a pin on the face of a circular plate rotating about its centre of figure (Article 63, Goodeve). Show the mechanism in three positions.

Problem 20. Construct a plate cam that will give to the centre of a roller bearing upon it, and moving on a straight line intersecting and perpendicular to the axis of the cam, a uniform upward motion of $3\frac{3}{8}$ inches for its first one-half revolution, and a uniform downward motion of $3\frac{3}{8}$ inches for its second one-half revolution. The smallest radius of the cam to be $2\frac{3}{16}$ inches, the diameter of the roller to be two inches. Make drawing, full size.

Problem 21. Construct a plate cam, which by turning left handed will give in the first one-half revolution a uniform upward motion of three inches, to the centre of a roller two inches in diameter, and in the remaining one-half revolution a uniform downward motion of three inches. The lowest position of the centre of the roller to be five inches from the axis of the cam, and the line of motion of the roller to fall three inches to the right of the axis of the cam.

Problem 22. Construct a plate cam to turn right handed, and give a reciprocatiug uniform motion of six inches to a piece in the plane of the cam plate, below the axis of the cam and twenty inches to the right. The smallest radius of the cam to be three inches and the largest radius five inches.

Problem 23. Construct a plate cam to give a uniform upward motion to a reciprocating piece and roller, of 3 inches during $\frac{4}{12}$ of a revolution, a uniform downward motion of 3 inches for $\frac{2}{12}$ of a revolution, and a period of rest for $\frac{6}{12}$ of a revolution. The line of direction to pass through the axis of the cam. The centre of the roller in its lowest position to be three inches from the centre of the cam. The roller is one inch diameter.

Problem 24. Construct a plate cam to give a uniformly accelerated motion for one-third of a revolution, a period of rest for one-third of a revolution, and to return instantly to its first position with reference to the axis, and then to remain at rest for the remainder of the revolution. Take the minimum radius as two inches, the maximum as four inches, and the line of direction of the motion to pass through the axis of the cam. Any accelerated motion properly constructed and explained will fulfill the stated conditions. Harmonic motion will accomplish this.

Problem 25. A cylindrical cam eight inches outside diameter, turning right handed as seen from the right, is to move uniformly a roller or slide, two inches to the right during one-fourth of a revolution, to rest one-fourth a revolution, two inches to the left during one-fourth a revolution, and to rest the remainder. Construct the development of the outline of the groove, the groove to be one inch wide.

Problem 26. Construct a cylindrical cam that in turning once right handed, shall give two complete oscillations to a piece connected with it by groove and roller. The cam to be eight inches in diameter, the extent of the entire oscillation two inches.

Problem 27. Construct a cam plate that oscillating through 60°, shall give a accelerating and retarded vertical motion of three inches to a two inch roller, the line of direction of the motion to pass three inches to the right of the axis of the cam, and the axis of the roller in its lowest position to be six inches from the axis of the cam. Explain the circumstances of motion, should the cam plate revolve on a shaft uniformly.

Problem 28. Make the construction for the following: Two shafts in parallel planes 7 feet apart, are to be connected by a quarter turn belt 3 inches wide, so as to cause the lower shaft, *i. e.*, the one perpendicular to the plane of the drawing to turn left handed. The upper shaft, which is the driver, is to make 200 revolutions, while the lower or follower makes 150 revolutions. The diameter of the pulley on the upper shaft is 9 inches, required the horizontal and 2 vertical projections of the system, neglecting the thickness of the belt, scale one-sixth.

Problem 29. Design a mechanism to register the number of rotations of the shaft of an engine (note Article 356, Figs. 341 and 289, Chapter 9, Goodeve). Take two cases, first a train of wheels and the single tooth wheel suggested in Fig. 289, Goodeve; second, follow the idea of Article 222, Goodeve, and the star wheel. Make a sketch showing a practicable attachment to the shaft of an engine.

Problem 30. Illustrate a manner of connection of a revolving shaft with another line of shafting whose direction is coincident, whereby the rotation of the first can be imparted to the second, either constantly, or at will. Look up friction clutches, etc., and see Fig. 311, Goodeve, for suggestion.

Problem 31. Having a train of wheels whose value is 194 to within less than 1 (the largest wheel has 90 and the smallest 12 teeth), construct a design for a hand-crane, using this train, a drum for rope 12 inches in diameter, crank levers to work same 20 inches long, and 60 lbs. applied power at the levers. Determine the number and size of the wheels, indicate their arrangement, and show the mechanical advantage of this combination, neglecting all resistances save the weight raised (See Goodeve's Principles of Mechanics, Fig. 78, Page 113).

Problem 32. Design a hand-crane to lift 6000 lbs. when 60 lbs. is applied to the crank handles. Taking the expression $Wr = ple$, the equation of work, we have $W = 6000$ lbs. $p = 60$ lbs. We have a train whose value $e = \frac{40}{3}$, r can be taken 6 inches, or any suitable dimension, whence l is determined. Find the number and dimensions (in teeth) of the wheels of the train the largest wheel to have 200, the smallest 30 teeth, and make a sketch showing the arrangement of the parts.

Problem 33. A certain train consists of 6 wheels, the largest has 40 and the smallest has 6 teeth, its value is 14 to within less than unity. If applied to a crane, the crank lever being 18″, the radius of the rope drum 6″, what pull will it exert on the rope if 30 lbs. be applied at the cranks. Make a sketch.

Problem 34. The distance from the axis of the trunnions of a gun weighing 3016 lbs. to the elevating screw is 3 feet, and the distance of the center of gravity of the gun from the same axis is 4 inches. If the distance between the threads of the screw be $\frac{2}{3}$ of an inch, and the length of the lever 5 inches, what power must be applied to sustain the gun in a horizontal position?

Problem 35. A center pivot carriage weighs 10,000 lbs. and turns on a pivot 30 inches in diameter; the top of the pivot is a spur gear 40 inches in diameter; with its perimeter gears a pinion, 5 inches in diameter; on the axis of this pinion is a worm wheel 10 inches in diameter with 30 teeth; the lever of the worm is 5 inches; one revolution of the worm advances the worm wheel 1 tooth. What force at the worm handle is necessary to move the carriage, neglecting all hurtful resistances except the friction on the pivot that supports the weight of the carriage?

Problem 36. A closed cylindrical vessel, the radius of whose base is 8 inches, is filled with water and placed with its axis horizontal. Find the pressure of the water against one end.

Problem 37. The depth of water in a vessel is 10 feet, the base of the vessel is a square with a side 18 inches long. What is the pressure on it?

Problem 38. A square $A\ B\ C\ D$ is immersed in a liquid at some depth, and turns about the horizontal side $A\ B$ as an axis. When the square is in the lowest position, hanging vertically, the pressure on its face is twice what it would be if the square were rotated about *A B*, so as to take its highest position. What is the depth of *A B*?

Problem 39. The pressure of a liquid on a square is ¼ the weight of a cube of the liquid whose edge is equal to a side of the square. If one edge of the square be in the surface of the fluid, what is the inclination of the square to the horizon?

Problem 40. Find the pressure on a circle 6 inches in diameter, when immersed at a depth of 1 mile below the surface of the sea; sea water weighing 64 lbs. to a cubic foot.

Problem 41. The depth of a dock gate is 32 feet and its breadth is 35 feet. Find the pressure on it when the water rises to a height of 20 feet on the outside.

Problem 42. In Fig. 155, Goodeve's Mechanics, $ec = 3\frac{1}{4}$ inches, $ea = 5$ inches, the valve weighs 68 lbs. What head of water would be held back by such a valve before the pressure could cause it to lift?

Problem 43. The breadth of a water passage closed by a *pair* of gates is 10 feet and its depth is 6 feet. The hinges are placed at 1 foot from top and bottom. Find the stress on the lower hinge when the water rises to the top of the gates on one side.

Problem 44. A piston moves through a cylinder 18″ in diameter, full of water, with a uniform velocity of 10 feet per second. What must be the area of the opening through the end of the cylinder through which the liquid must be forced, in order that the pressure on the piston may be 10,000 lbs. per square foot.

Problem 45. In a hydraulic press the plunger is ⅞ of an inch and the ram is 9¼ inches in diameter. The plunger is worked by a lever; the distance from the pump to the fulcrum is 3¼ inches, and that from the fulcrum to the power *P* is 78 inches. What is the gain in power?

Problem 46. In a hydraulic press the plunger is ¾ of an inch and the ram 10 inches in diameter, the arms of the lever are 6 feet and 1 foot. A weight of 20 lbs. is hung at the end of the lever. Find the pressure on the ram.

Problem 47. The plunger of a force pump is 8¾ inches in diameter, the length of the stroke is 30 inches, and the pressure of the water is 7200 lbs. per square foot. Find the number of units of work done in one stroke.

Problem 48. The cross section of a sucking and forcing pump is 6 square feet, the play of the piston 3 feet, and the height of the point of delivery 50 feet above the reservoir. What must be the horse power of an engine to give 30 double strokes per minute, hurtful resistances being neglected?

Problem 49. In a hydrostatic press the areas of the pistons are 2 and 400 square inches, and the arms of the lever are 1 and 20 inches. Required the pressure on the larger piston for each pound of pressure on the longer arm of the lever.

Problem 50. The areas of the piston of a hydrostatic press are 3 and 300 square inches, and the shorter arm of the lever is one inch. What must be the length of the longer arm, that a force of 1 lb. may produce a pressure of 1000 lbs.?